

Matrix Representation of Intuitionistic Fuzzy Soft Graph

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ABSTRACT:

This paper introduces intuitionistic fuzzy soft graph matrix as a powerful tool for modeling and addressing uncertainty in a wide range of scientific and practical fields. By representing intuitionistic fuzzy soft graphs in matrix form, the computational efficiency is improved when dealing with uncertain or imprecise situations. This paper defines several key operations that can be performed on intuitionistic fuzzy soft graph matrices, such as union, intersection, complement and some other operations evaluates their usefulness and relevance in decision making process.

Keywords: Intuitionistic Fuzzy Soft Graph, Intuitionistic Fuzzy Soft Graph Matrix, Matrix Operations.

1 Introduction

Soft set theory, introduced by Molodtsov [1], provides a parameterized approach to uncertainty without the need for additional constraints on the data. Subsequently, Maji et al. [2] explored the algebraic properties and applications of soft sets. On the other hand, fuzzy set theory, proposed by Zadeh [3], models uncertainty using degrees of membership, but it does not explicitly account for non-membership or hesitation. To address this drawback, Atanassov [4] introduced intuitionistic fuzzy sets, characterized by both membership and non-membership functions, allowing a more expressive representation of uncertainty. The integration of fuzzy concepts with soft sets led to the development of fuzzy soft sets and intuitionistic fuzzy soft sets [5]. Parallel to these

developments, graph theory has played a vital role in modeling relational structures in complex systems. Fuzzy graphs and intuitionistic fuzzy graphs were introduced to incorporate uncertainty into graph structures [6–9]. To further enhance modeling flexibility, Akram et al. and later Shyla and Mathew Varkey introduced intuitionistic fuzzy soft graphs (IFSGs), which represent a parameterized family of intuitionistic fuzzy graphs [13].

Although intuitionistic fuzzy soft graphs provide a powerful theoretical framework, their direct application to real-world problems can be computationally challenging due to the complexity of handling multiple parameters and uncertainty measures. Matrix representations offer a natural and

efficient way to store, manipulate, and analyze graph-based information. Matrix-based approaches have been successfully applied to classical graphs, fuzzy graphs, and soft graphs, significantly improving computational efficiency and analytical clarity.

Motivated by these advantages, the present work introduces a matrix representation of intuitionistic fuzzy soft graphs, referred to as the intuitionistic fuzzy soft graph matrix (IFSGM). This matrix formulation systematically captures the membership, non-membership, and hesitation degrees of vertices and edges under each parameter, providing a compact and structured representation. Several fundamental operations on IFSG matrices—including union, intersection, complement, Cartesian product, dot product, max–min and min–max products, arithmetic mean, and geometric mean—are formally defined and illustrated with examples.

2 Preliminaries

Definition 2.1:[6]

An Intuitionistic Fuzzy Graph (IFG) is of the form $G = (V, E)$ said to be a minmax IFG if

- (1) $V = \{v_1, v_2, \dots, v_n\}$ such that $\mu_i : V \rightarrow [0,1]$ and $\nu_i : V \rightarrow [0,1]$, denotes the degree of membership and non-membership of an element $v_i \in V$ respectively and $0 \leq \mu_i + \nu_i \leq 1$, for every $v_i \in V$,
- (2) $E \subseteq V \times V$ where $\mu_{ij} : V \times V \rightarrow [0,1]$ and $\nu_{ij} : V \times V \rightarrow [0,1]$ are such that

$$\begin{aligned}\mu_{ij} &\leq \min(\mu_i, \mu_j), \\ \nu_{ij} &\leq \max(\nu_i, \nu_j)\end{aligned}$$

denotes the degree of membership and non-membership of an edge $e_{ij} = (v_i, v_j) \in E$ respectively, where, $0 \leq \mu_{ij} + \nu_{ij} \leq 1$, for every $e_{ij} = (v_i, v_j) \in E$. The degree of hesitance (hesitation degree) of the vertex $v_i \in V$ in G is $\Pi_i = 1 - \mu_i - \nu_i$ and the degree of hesitance (hesitation degree) of an edge $e_{ij} = (v_i, v_j) \in E$ in G is $\Pi_{ij} = 1 - \mu_{ij} - \nu_{ij}$

Definition 2.2:[13]

Let U be an initial universal set, P be a set of parameters, $\mathcal{P}(U)$ be the power set of U and $A \subseteq P$. A pair (F, A) is called an intuitionistic fuzzy soft set over U , where F is a mapping given by $F : A \rightarrow \mathcal{P}(U)$.

Definition 2.3:[15]

An intuitionistic fuzzy soft graph (IFSG) G on a nonempty set V of G^* is an ordered 3-tuple $G = (\phi, \psi, A)$ such that,

(i) A is a non-empty set of parameters, where $A \subseteq V$

(ii) (ϕ, A) is an intuitionistic fuzzy soft set over V ,

(iii) (ψ, A) is an intuitionistic fuzzy soft relation on V , i.e., $\psi : A \rightarrow \mathcal{P}(V \times V)$ where, $\mathcal{P}(V \times V)$ is intuitionistic fuzzy power set,

(iv) $H(a_i) = (\phi(a_i), \psi(a_i))$ is an intuitionistic fuzzy subgraph, for all $a_i \in A$.

where,

$$\begin{aligned}\phi_\mu(a_i)(uv) &\leq \min(\phi_\mu(a_i)(u), \phi_\mu(a_i)(v)), \\ \psi_\nu(a_i)(uv) &\leq \max(\psi_\nu(a_i)(u), \psi_\nu(a_i)(v))\end{aligned}$$

such that $\phi_\mu(a_i)(uv) + \psi_\nu(a_i)(uv) \leq 1, \forall a_i \in A, u, v \in V$.

Note that, $\phi_\mu(a_i)(uv) = \psi_\nu(a_i)(uv) = 0, \forall uv \in (V \times V) - E, a_i \in A$.

Where, (ϕ, A) is called an intuitionistic fuzzy soft vertex and (ψ, A) is called an intuitionistic fuzzy soft edge. Thus, $((\phi, A), (\psi, A))$ is called an intuitionistic fuzzy soft graph.

Furthermore, throughout this paper, $G^* = (V, E)$ be a underlying crisp graph. $G = (\phi, \psi, A) = ((\phi, A), (\psi, A))$ is an intuitionistic fuzzy soft graph which is a parameterized family of intuitionistic fuzzy subgraphs. i.e., $G = \{H(a_i) | \forall a_i \in A\}$

Example 2.4:

Let $G^* = (V, E)$ be a simple crisp graph, where, $V = \{v_1, v_2, v_3, v_4\}$ and $E = \{e_1, e_2, e_3, e_4, e_5\}$. Consider the parameter set $A = \{a_1, a_2\}$ and let (ϕ, A) be an IFSS over V and $\phi : A \rightarrow \mathcal{P}(V)$ is defined by

$$\begin{aligned}\phi(a_1) &= \{(v_1, 0.3, 0.4), (v_2, 0.5, 0.2), (v_3, 0.7, 0.1), (v_4, 0.9, 0.0)\}, \\ \phi(a_2) &= \{(v_1, 0.7, 0.1), (v_2, 0.3, 0.4), (v_3, 0.4, 0.4), (v_4, 0.6, 0.2)\}\end{aligned}$$

in Fig 2.1

Let (ψ, A) be an IFSS over E and $\psi : A \rightarrow \mathcal{P}(E)$ is defined by

$$\begin{aligned}\psi(a_1) &= \{(e_2, 0.6, 0.1), (e_3, 0.3, 0.3), (e_5, 0.4, 0.1)\}, \\ \psi(a_2) &= \{(e_1, 0.2, 0.3), (e_2, 0.4, 0.2), (e_4, 0.3, 0.3), (e_5, 0.1, 0.3)\}\end{aligned}$$

in Fig 2.1

Clearly, $H(a_1) = (\phi(a_1), \psi(a_1))$, and $H(a_2) = (\phi(a_2), \psi(a_2))$ are intuitionistic fuzzy subgraphs corresponding to $a_1, a_2 \in A$ respectively. Hence $G = \{H(a_1), H(a_2)\}$ is an IFSG of G^* .

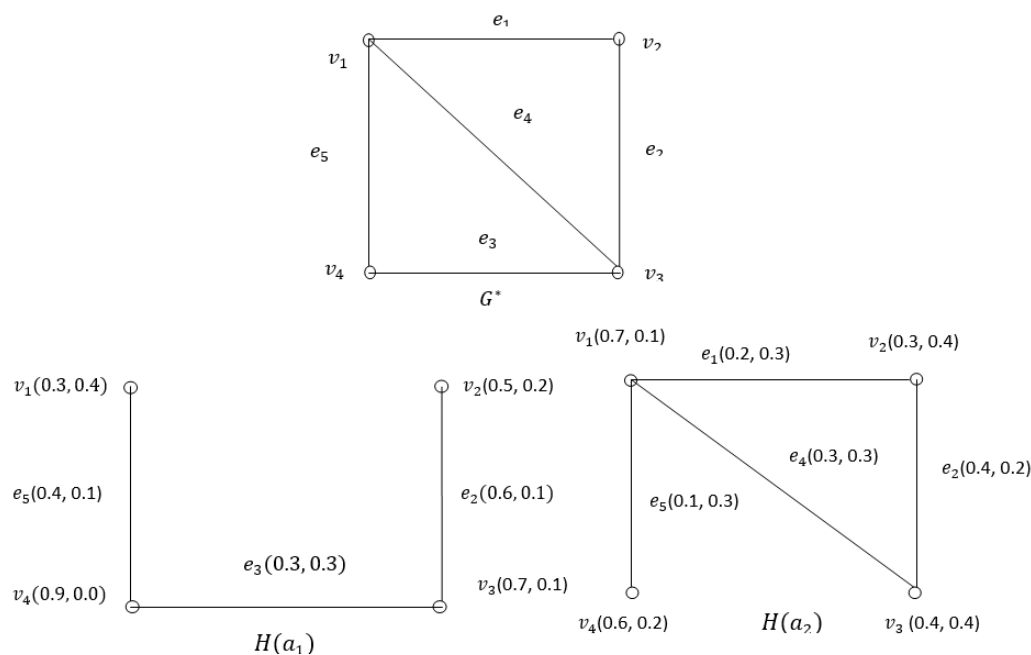


Figure 2.1

Definition 2.5:[13]

Let G_1 and G_2 be two IFSGs. The union of G_1 and G_2 is an IFSG denoted by $G = G_1 \cup G_2 = (\phi, \psi, A \cup B)$, where $(\phi, A \cup B)$ is an IFSS over $V_1 \cup V_2$, $(\psi, A \cup B)$ is an IFSS over $E_1 \cup E_2$ and $H = (\phi, \psi, A \cup B)$ are IFSGs for all $a \in A \cup B$ defined by

$$H(a) = \begin{cases} H_1(a) & \text{if } a \in A - B \\ H_2(a) & \text{if } a \in B - A \\ H_1(a) \cup H_2(a) & \text{if } a \in A \cap B \end{cases}$$

Where, $H_1(a) \cup H_2(a)$ denotes the union of two IFSGs.

Definition 2.6:[13]

The complement of an IFSG $G = (\phi, \psi, A)$ denoted by $G^c = (\phi^c, \psi^c, A^c)$ is defined by $M^c = M, \phi^c(a) = \phi(a), \psi^c(a)(u, v) = \phi_\mu(a)(u) \wedge \phi_\mu(a)(v) - \psi_\mu(a)(u, v)$ and $\psi^c(a)(u, v) = \phi_\nu(a)(u) \wedge \phi_\nu(a)(v) - \psi_\nu(a)(u, v)$ for all $u, v \in V, a \in A$.

Definition 2.7:[5]

Let $G = (V, E)$ be an intuitionistic fuzzy graph where $V = \{v_1, v_2, \dots, v_n\}$. The incidence matrix of an intuitionistic fuzzy graph G is $B = \{< b_{\mu_{ij}}, b_{\nu_{ij}} >\}_{n \times m}$, where n and m represents

the number of vertices and number of edges of G respectively, whose entries of B are as follows:

$$B = \{< b_{\mu_{ij}}, b_{\nu_{ij}} >\}_{n \times m} = \begin{cases} < \mu(e_j), \nu(e_j) >, & \text{if vertex } v_i \text{ is incident to the edge } e_j \\ < 0, 1 >, & \text{otherwise} \end{cases}$$

3 Intuitionistic Fuzzy Soft Graph Matrix**Definition 3.1:**

The incidence matrix M_{IFSG} of the intuitionistic fuzzy subgraph of G^* is defined as

$$M_{IFSG[H(a_i)]} = \begin{cases} < \mu(e_j), \nu(e_j) >, & \text{if vertex } v_i \text{ is incident to the edge } e_j \\ < 0, 1 >, & \text{otherwise} \end{cases}$$

The matrix $M_{IFSG[H(a_i)]}$ is designated as the Intuitionistic Fuzzy Soft Graph Matrix for each $a_i \in A$.

Remark:

M_{IFSG} denotes the set of all matrices related to the intuitionistic fuzzy soft graph of G , for each $a \in A$.

Example 3.2:

$M_{IFSG[H(a_1)]}$ and $M_{IFSG[H(a_2)]}$ of G given in example 2.4. are,

$$M_{IFSG[H(a_1)]} = \begin{bmatrix} \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a_2)]} = \begin{bmatrix} \langle 0.2,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0.1,0.3 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.4,0.2 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.4,0.2 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.1,0.3 \rangle \end{bmatrix}$$

4 Operations on Fuzzy Soft Graph Matrix

Definition 4.1:

Let $M_{IFSG[H(a_1)]} = [m_{ij}]_{n \times m}$ and $M_{IFSG[H(a_2)]} = [n_{ij}]_{n \times m}$ be two intuitionistic fuzzy soft graph matrices of G , where, $a_1, a_2 \in A$. The union of the two intuitionistic fuzzy soft graph matrices is defined as follows:

$$M_{IFSG[H(a_1)]} \cup M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} = [l_{ij}]_{n \times m}$$

where,

$$l_{ij} = \langle \max\{\mu_i, \mu_j\}, \min\{\nu_i, \nu_j\} \rangle \quad \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

Example 4.2:

From the example 3.2 we have,

$$M_{IFSG[H(a_1)]} = \begin{bmatrix} \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a_2)]} = \begin{bmatrix} \langle 0.2,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0.1,0.3 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.4,0.2 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.4,0.2 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.1,0.3 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0.2,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0.4,0.1 \rangle \\ \langle 0.2,0.3 \rangle & \langle 0.6,0.1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0.3,0.3 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \end{bmatrix}$$

Definition 4.3:

Let $M_{IFSG[H(a_1)]} = [m_{ij}]_{n \times m}$ and $M_{IFSG[H(a_2)]} = [n_{ij}]_{n \times m}$ be two intuitionistic fuzzy soft graph matrices of G , where, $a_1, a_2 \in A$. The intersection of the two intuitionistic fuzzy soft graph matrices is defined as follows:

$$M_{IFSG[H(a_1)]} \cap M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} = [l_{ij}]_{n \times m}$$

where,

$$l_{ij} = \langle \min\{\mu_i, \mu_j\}, \max\{\nu_i, \nu_j\} \rangle \quad \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m.$$

Example 4.4:

From the example 3.2 we have,

$$M_{IFSG[H(a_1)]} = \begin{bmatrix} \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.6,0.1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.3,0.3 \rangle & \langle 0,1 \rangle & \langle 0.4,0.1 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a_2)]} = \begin{bmatrix} \langle 0.2, 0.3 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.1, 0.3 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.1, 0.3 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a)]} = \begin{bmatrix} M_{IFSG[H(a_1)]} \cap M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.1, 0.3 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.1, 0.3 \rangle \end{bmatrix}$$

Definition 4.5:

Let $M_{IFSG[H(a)]} = [m_{ij}]_{n \times m}$ be the intuitionistic fuzzy soft graph matrix of G for $a \in A$. The complement of M_{IFSG} is defined as,

$$M_{IFSG[H(a)]}^c = M_{IFSG[H(a)]}^c = [l_{ij}]_{n \times m}$$

Where, $l_{ij} = \langle v(e_j), \mu(e_i) \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

4.1 Some Other Operators on Intuitionistic Fuzzy Graph Matrices

Let $M_{IFSG[H(a_1)]} = [m_{ij}]_{n \times m}$ and $M_{IFSG[H(a_2)]} = [n_{ij}]_{n \times m}$ be two intuitionistic fuzzy soft graph matrices of G , where $a_1, a_2 \in A$.

i) Cartesian Product:

$$M_{IFSG[H(a_1)]} \times M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} \\ = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \{\mu_i \cdot \mu_j\}, \{v_i \cdot v_j\} \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

ii) Dot Product:

$$M_{IFSG[H(a_1)]} \cdot M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} \\ = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \{(\mu_i \cdot \mu_j), (v_i + v_j - v_i \cdot v_j)\} \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

iii) Arithmetic Mean:

$$M_{IFSG[H(a_1)]} \dot{+} M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} \\ = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \left\{ \frac{\mu_i + \mu_j}{2}, \frac{v_i + v_j}{2} \right\} \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

iv) Geometric Mean:

$$M_{IFSG[H(a_1)]} \dot{\div} M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]} \\ = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \{ \sqrt{\mu_i \cdot \mu_j}, \sqrt{v_i \cdot v_j} \} \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

Example 4.6:

From the example 3.2 we have,

$$M_{IFSG[H(a_1)]} = \begin{bmatrix} \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0, 1 \rangle & \langle 0.6, 0.1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.6, 0.1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0, 1 \rangle & \langle 0.4, 0.1 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0.1, 0.3 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a_2)]} = \begin{bmatrix} \langle 0.2, 0.3 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0.2, 0.3 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.4, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0.3, 0.3 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.1, 0.3 \rangle \end{bmatrix}$$

Cartesian Product:

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.04, 0.03 \rangle \\ \langle 0, 1 \rangle & \langle 0.24, 0.02 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.24, 0.02 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.04, 0.03 \rangle \end{bmatrix}$$

Dot Product:

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.04, 0.37 \rangle \\ \langle 0,1 \rangle & \langle 0.24, 0.28 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.24, 0.28 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.04, 0.37 \rangle \end{bmatrix}$$

Arithmetic Mean:

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0.1, 0.65 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.15, 0.65 \rangle & \langle 0.25, 0.2 \rangle \\ \langle 0.1, 0.65 \rangle & \langle 0.5, 0.15 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.5, 0.15 \rangle & \langle 0.15, 0.65 \rangle & \langle 0.15, 0.65 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.15, 0.65 \rangle & \langle 0,1 \rangle & \langle 0.25, 0.2 \rangle \end{bmatrix}$$

Geometric Mean:

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.04, 0.03 \rangle \\ \langle 0,1 \rangle & \langle 0.24, 0.02 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0.24, 0.02 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle \\ \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0,1 \rangle & \langle 0.04, 0.03 \rangle \end{bmatrix}$$

Theorem 4.7:

The union of two intuitionistic fuzzy soft graph matrices is also a intuitionistic fuzzy soft graph matrix.

Proof:

Let $G = (\phi, \psi, A)$ be a intuitionistic fuzzy soft graph over a simple graph $G^* = (V, E)$. M_{IFSG} be a set of all matrices of intuitionistic fuzzy soft graph associated with G .

Consider two intuitionistic fuzzy soft graph matrices

$$M_{IFSG[H(a_1)]} = [m_{ij}]_{n \times m} \text{ and } M_{IFSG[H(a_2)]} = [n_{ij}]_{n \times m}$$

where $a_1, a_2 \in A$.

$$[m_{ij}]_{n \times m} = \langle \mu_{ij}(a_1), \nu_{ij}(a_1) \rangle \quad \text{and}$$

$$[n_{ij}]_{n \times m} = \langle \mu_{ij}(a_2), \nu_{ij}(a_2) \rangle$$

The union of the two intuitionistic fuzzy soft graph matrices is,

$$M_{IFSG[H(a_1)]} \cup M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]}$$

$$M_{IFSG[H(a)]} = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \max\{\mu_i, \mu_j\}, \min\{\nu_i, \nu_j\} \rangle$
 $\forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

Since, m_{ij} and n_{ij} are intuitionistic fuzzy pairs, we have $0 \leq \mu_{ij}(a) + \nu_{ij}(a) \leq 1$.

Therefore, $\mu_{ij}(a) + \nu_{ij}(a) = \max\{\mu_i, \mu_j\} + \min\{\nu_i, \nu_j\} \leq 1$

which implies that each l_{ij} is an intuitionistic fuzzy value.

Thus, $M_{IFSG[H(a)]}$ satisfies all the conditions of an intuitionistic fuzzy soft graph matrix.

Hence, the union of two intuitionistic fuzzy soft graph matrices is also a intuitionistic fuzzy soft graph matrix.

Theorem 4.8:

The intersection of two intuitionistic fuzzy soft graph matrices is also a intuitionistic fuzzy soft graph matrix.

Proof:

Let $G = (\phi, \psi, A)$ be a intuitionistic fuzzy soft graph over a simple graph $G^* = (V, E)$. M_{IFSG} be a set of all matrices of intuitionistic fuzzy soft graph associated with G .

Consider two intuitionistic fuzzy soft graph matrices

$$M_{IFSG[H(a_1)]} = [m_{ij}]_{n \times m} \text{ and } M_{IFSG[H(a_2)]} = [n_{ij}]_{n \times m}$$

where $a_1, a_2 \in A$.

$$[m_{ij}]_{n \times m} = \langle \mu_{ij}(a_1), \nu_{ij}(a_1) \rangle \quad \text{and}$$

$$[n_{ij}]_{n \times m} = \langle \mu_{ij}(a_2), \nu_{ij}(a_2) \rangle$$

The intersection of the two intuitionistic fuzzy soft graph matrices is,

$$M_{IFSG[H(a_1)]} \cap M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]}$$

$$M_{IFSG[H(a)]} = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \min\{\mu_i, \mu_j\}, \max\{\nu_i, \nu_j\} \rangle$
 $\forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

Since, m_{ij} and n_{ij} are intuitionistic fuzzy pairs, we have $0 \leq \mu_{ij}(a) + \nu_{ij}(a) \leq 1$.

Therefore, $\mu_{ij}(a) + \nu_{ij}(a) = \min\{\mu_i, \mu_j\} + \max\{\nu_i, \nu_j\} \leq 1$

which implies that each l_{ij} is an intuitionistic fuzzy value.

Thus, $M_{IFSG[H(a)]}$ satisfies all the conditions of an intuitionistic fuzzy soft graph matrix.

Hence, the intersection of two intuitionistic fuzzy soft graph matrices is also a intuitionistic fuzzy soft graph matrix.

Theorem 4.9:

The complement of intuitionistic fuzzy soft graph matrix is also a intuitionistic fuzzy soft graph matrix.

Proof:

Let $G = (\phi, \psi, A)$ be a intuitionistic fuzzy soft graph over a simple graph $G^* = (V, E)$. M_{IFSG} be a set of all matrices of intuitionistic fuzzy soft graph associated with G .

Consider an intuitionistic fuzzy soft graph matrices

$M_{IFSG[H(a)]} = [m_{ij}]_{n \times m}$, where $a \in A$.

$$m_{ij} = \langle \mu_{ij}(a), \nu_{ij}(a) \rangle$$

The intuitionistic fuzzy soft graph matrix's complement is,

$$M_{IFSG[H(a)]}^c = M_{IFSG[H(a)]}^c$$

$$M_{IFSG[H(a)]}^c = [l_{ij}]_{n \times m}$$

where, $l_{ij} = \langle \nu_{ij}(a), \mu_{ij}(a) \rangle \forall i = 1, 2, \dots, n, j = 1, 2, \dots, m$.

Since, m_{ij} is an intuitionistic fuzzy pairs, we have $0 \leq \mu_{ij}(a) + \nu_{ij}(a) \leq 1$.

Therefore, $\nu_{ij}(a) + \mu_{ij}(a) \leq 1$

Thus, $M_{IFSG[H(a)]}^c$ satisfies all the conditions of an intuitionistic fuzzy soft graph matrix.

Hence, the complement of intuitionistic fuzzy soft graph matrix is also a intuitionistic fuzzy soft graph matrix

5 Application

A parcel delivery company transports goods between cities. Some cities are directly

connected, while others require intermediate routes. Due to uncertainty in real-world conditions, such as varying delivery costs and customer satisfaction levels, a precise evaluation of delivery routes is difficult. Hence, an intuitionistic fuzzy soft graph framework is employed to model this uncertainty and support decision-making.

Let $G^* = (V, E)$ be a simple graph, where $V = \{v_1, v_2, v_3, v_4\}$ represent cities and $E = \{e_1, e_2, e_3, e_4, e_5\}$ represents delivery routes between cities.

Consider the parameter set $A = \{a_1, a_2\}$, where parameters a_1 denotes delivery cost and a_2 denotes customer satisfaction.

Let (ϕ, A) be an IFSS over V defined by,

$$\phi(a_1) = \{(v_1, 0.3, 0.4), (v_2, 0.5, 0.2), (v_3, 0.7, 0.1), (v_4, 0.9, 0.0)\},$$

$$\phi(a_2) = \{(v_1, 0.7, 0.1), (v_2, 0.3, 0.4), (v_3, 0.4, 0.4), (v_4, 0.6, 0.2)\},$$

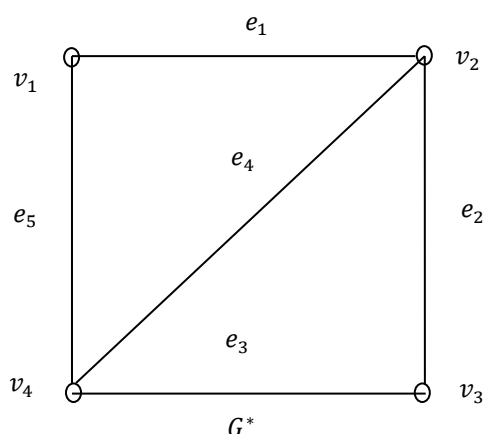
Let (ψ, A) be an IFSS over E defined by

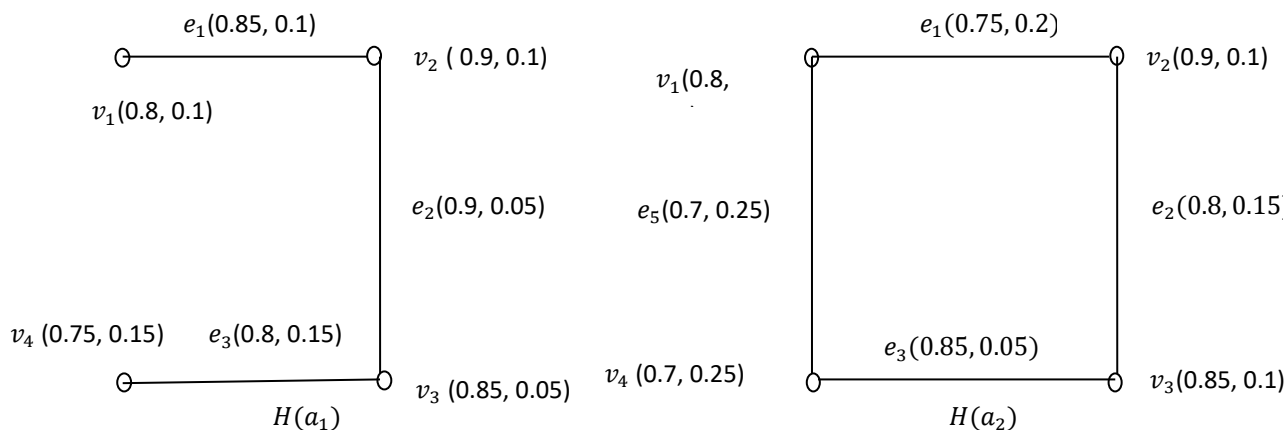
$$\psi(a_1) = \{(e_2, 0.6, 0.1), (e_3, 0.3, 0.3), (e_5, 0.4, 0.1)\},$$

$$\psi(a_2) = \{(e_1, 0.2, 0.3), (e_2, 0.4, 0.2), (e_4, 0.3, 0.3), (e_5, 0.1, 0.3)\},$$

Clearly, $H(a_1) = (\phi(a_1), \psi(a_1))$, and $H(a_2) = (\phi(a_2), \psi(a_2))$ are IFGs corresponding to the parameters a_1 and a_2 respectively as shown in figure below.

Hence $G^* = \{H(a_1), H(a_2)\}$ is an IFSG of G .





$$M_{IFSG[H(a_1)]} = \begin{bmatrix} \langle 0.85, 0.1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0.85, 0.1 \rangle & \langle 0.9, 0.05 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.9, 0.05 \rangle & \langle 0.75, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.75, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \end{bmatrix}$$

$$M_{IFSG[H(a_2)]} = \begin{bmatrix} \langle 0.75, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.7, 0.25 \rangle \\ \langle 0.75, 0.2 \rangle & \langle 0.8, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.8, 0.15 \rangle & \langle 0.85, 0.05 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.85, 0.05 \rangle & \langle 0, 1 \rangle & \langle 0.7, 0.25 \rangle \end{bmatrix}$$

Decision-Making Procedure

When making parcel delivery decisions, the company aims to minimize delivery cost and maximize customer satisfaction simultaneously. To achieve this, the intersection of the intuitionistic fuzzy soft graph matrices is computed:

$$M_{IFSG[H(a_1)]} \cap M_{IFSG[H(a_2)]} = M_{IFSG[H(a)]}$$

The intersection operation ensures that only those routes with lower cost (higher membership in a_1) and higher customer satisfaction (higher membership in a_2) are retained, while preserving the intuitionistic fuzzy constraints.

$$M_{IFSG[H(a)]} = \begin{bmatrix} \langle 0.75, 0.2 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0.75, 0.2 \rangle & \langle 0.8, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0.8, 0.15 \rangle & \langle 0.75, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \\ \langle 0, 1 \rangle & \langle 0, 1 \rangle & \langle 0.75, 0.15 \rangle & \langle 0, 1 \rangle & \langle 0, 1 \rangle \end{bmatrix}$$

Result and Interpretation

From the resulting matrix $M_{IFSG[H(a)]}$, the delivery paths

$V_1 \rightarrow V_2, V_2 \rightarrow V_3, V_3 \rightarrow V_4$ emerge as optimal routes. These paths correspond to delivery routes that jointly exhibit lower transportation cost and higher customer satisfaction under uncertainty.

Thus, the proposed intuitionistic fuzzy soft graph matrix framework provides a systematic and flexible decision-support mechanism for selecting efficient delivery routes in uncertain logistics networks.

Unlike classical fuzzy or intuitionistic fuzzy graph models, the proposed intuitionistic fuzzy soft graph matrix enables simultaneous evaluation of multiple parameters within a single algebraic framework, thereby offering greater flexibility in decision-making under uncertainty.

6 Conclusion

In this paper, a matrix representation of intuitionistic fuzzy soft graphs has been developed. Fundamental matrix operations such as union, intersection, and complement were formally defined. The proposed intuitionistic fuzzy soft graph matrix framework provides a structured algebraic foundation for decision-

making in uncertain environments. Future research may focus on developing algorithmic decision-making procedures, complexity analysis, and real-world implementations in areas

such as logistics networks, social systems, and optimization problems.

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