Enforcement Of M-Fuzzy Off- Group with Respect to Fuzzy Over Set and Fuzzy Under Set –A New Approach

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HOW TO CITE:
R.Nagarajan, P.K.Eswari, S.Elango, K.Balamurugan (2026). Enforcement Of M-Fuzzy Off- Group with Respect to Fuzzy Over Set and Fuzzy Under Set –A New Approach. International Journal of Special Education, 41(16s), 1320-1326

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ABSTRACT:
In this article, we investigate novel algebraic structures known as the M-fuzzy off-group and exposure an in-depth study of its fundamental mathematical properties. We investigate M-fuzzy over group and M-fuzzy under group and its properties. We believe that this new frame work will further in advance research in group theory and fuzzy modeling with offsets, and that it will open up new path of directions for various applications.

Keywords: fuzzy set, fuzzy group, fuzzy off set, fuzzy over set, fuzzy under set, level, M-fuzzy off-group.

1. Introduction

The concept of fuzzy sets was initiated by Zadeh [17]. Then it has become a vigorous area of research in analysis, topology, probability, statistic and measure theory etc. In [13] Rosenfeld used this concept to develop the theory of fuzzy groups. In fact, many basic properties in group theory are found to be carried over to fuzzy groups. Aktas and Cagman [1] studied the product of fuzzy subgroups and t – level subgroup. [2] Aktas discussed fuzzy quotient groups. Several mathematicians have followed the Rosenfeld approach in investigating the fuzzy subgroup theory. Fuzzy normal subgroups were studied by Wu [16] Kumar [7]. The concept of upper Q- fuzzy index order with upper Q – fuzzy subgroups studied by Nagarajan and Solairajn [4,9] a fuzzy subgroups of finite abelian groups, fuzzy subgroups of finite cyclic groups and fuzzy subgroup of symmetric groups S2, S3 and alternating group A4 was studied by some authors [11,14,12], N.Jacobson [5] introduced the concept of m – groups, m – subgroups. Some authors [3, 6, 8, 10] discussed the homomorphism and anti – homomorphism of fuzzy subgroups and give some properties for this subject.Fuzzy handling frame works such as fuzzy sets, rough sets, intuitionistic fuzzy sets, neutrosophic sets, picture fuzzy sets, hyper neutrosophic sets and pathogenic sets have

attracted sustained research interest. These frame works have been widely applied across various mathematical disciplines, including graph theory, topology, algebra and especially in group theory. A Brief Study on Fuzzy Off-Group Theory [15.].More recently, the idea of offset has emerged as a powerful and promising generalization of conventional fuzzy model. In this paper, we investigate novel algebraic structures known as the M-fuzzy off-group and exposure an in-depth study of its fundamental mathematical properties. We investigate M-fuzzy over group and M-fuzzy under group and its properties. We believe that this new frame work will further in advance research in group theory and fuzzy modeling with offsets, and that it will open up new path of directions for various applications.

2. Preliminaries

Definition-2.1: Fuzzy set: Let X be a non-empty set. A fuzzy set A drawn from X is defined as

$A = \{(x, \mu_A(x))/x \in X\}$, where $\mu_A: X \to [0,1]$ is the member function of the fuzzy set A.

Definition-2.2: Fuzzy Relation: A fuzzy set 'A' in a non-empty universe Y is a mapping

$A: Y \to [0,1]$. A fuzzy relation on Y is a fuzzy subset 'δ' is a fuzzy relation $Y \times Y$. If 'A' is a fuzzy set in 'Y' and 'δ' is a fuzzy relation on Y, then 'δ' is called a fuzzy relation on 'A' if $\delta(y,z) \leq min\{A(y), A(z)\}$, for all $y, z \in Y$.

Definition-2.3: Fuzzy off set: The concept of a fuzzy off set extends the classical fuzzy set by allowing membership degrees outside the unit interval $[0,1]$. Although this paper focuses on offsets, one can similarly define fuzzy over sets (all memberships ≥ 0 with some > 1) and fuzzy under sets (all memberships ≤ 1 with some < 0) using same principle. The related definitions such as neutrosophic off set are also well known.

Example-2.4: Let $Q = [20, 45]$ be the range of room temperatures$(C°)$. Define the uncertainty set 'A' representing " Comfortable" temperature by

$$A(x) = \begin{cases} 0, & x \leq 23 \\ \dfrac{x-23}{4}, & 23 < x < 35 \\ 1, & 35 \leq x \leq 38 \\ \dfrac{45-x}{5}, & 38 \leq x \leq 45 \\ 0, & x \geq 45 \end{cases}$$

Here, $A(x)$ measures the degree to which a temperature 'x' is regarded as comfortable. This uncertainty concept guides the HVAC control system to adjust heating or cooling gradually rather than using a simple on/off threshold.

Definition-2.5: Let X be a universe of discourse. A fuzzy off set '$\tilde{A}$' on X is a fuzzy set whose membership function $J_{\tilde{A}}: X \to [\Delta, \delta]$ with $\Delta < 0$ and $\delta > 1$. In particular, there must be elements $x, y \in X$ such that $J_{\tilde{A}}(x) > 1$ and $J_{\tilde{A}}(y) < 0$.

Example-2.6: Academic performance refers to a student level of achievement is educational tasks, typically measured through grades, tests and evaluations.

Let $X = \{\text{Ram}, \text{John}, \text{Gopi}, \text{Balu}\}$ be a set of students. After including extra benefit and punishments, each student's adjusted exam score 'r' may lie in $[-15,160]$. Define a membership function $J_{\tilde{A}}: X \to [-0.15,1.6]$, $J_{\tilde{A}}(\text{students}) = \frac{r}{100}$

Concretely, $J_{\tilde{A}}(\text{Ram}) = \frac{130}{100} = 1.3$, $J_{\tilde{A}}(\text{John}) = \frac{90}{100} = 0.9 J_{\tilde{A}}(\text{Gopi}) = \frac{75}{100} = 0.75$,

$J_{\tilde{A}}(\text{Balu}) = \frac{-15}{100} = -0.15$.Here, $J_{\tilde{A}}(\text{Ram}) > 1$ models over-achievement (extra benefit) and $J_{\tilde{A}}(\text{Balu}) < 0$ models a severe punishment. Therefore $\tilde{A}$ is a fuzzy offset on X.

Definition-2.7: Let M and G be a set and group respectively. A mapping $J: M \times G \to [\Delta, \delta]$

is called M-fuzzy offset in G. Let X be a set of measurement points in electrical circuits, whose each point 'x' has a measured voltage V_x . Fix a nominal voltage $V_{nom} = 3V$ and tolerance $\xi = 2V$. Define the membership function $J_{\tilde{A}}(\text{mx}) = 1 + \frac{V_x - V_{nom}}{\xi}$. Then, $J_{\tilde{A}}(\text{mx}) > 1$, if $V_x >$

$V_{nom} + \xi$ (over-voltage), $J_{\tilde{A}}(\text{mx}) < 0$, if $V_x < V_{nom} - \xi$ (under-voltage).

Then, $J: M \times G \to [\Delta, \delta]$ with $\Delta < 0 < 1 < \delta$ captures both extreme under and over voltage conditions as M-fuzzy offset.

Definition-2.8: A triangular norm (abbreviation t-norm) is a binary operation T on the interval [0,1] satisfying the following conditions:

(T1) T(x, y)=T(y, x) (commutativity)

(T2) T(x, T(y, z))=T(T(x, y),z) (associativity)

(T3) y ≤ z ⟹T(x, y)≤T(x, z) (monotonicity)

(T4) T(x, 1)=x (neutral element 1)

Examples-2.9

*TM(x, y)=min(x, y) (minimum or Gödel t-norm)

*TP(x, y)=x· y (product t-norm)

*TL(x, y)=max(x+y−1,0) (Lukasiewicz t-norm)

In this section, fuzzy group theory generalizes classical group theory by allowing membership degrees to group elements, capturing fuzzy and partial belonging. Related ideas such as the neutrosophic group are also well known. The concept of a fuzzy subgroup of a group is given below.

Definition-2.10: Fuzzy subgroup: Let $(G, ., e)$be a group with identity element e. A fuzzy subset of G is a function $J: G \to [0,1]$. Such a fuzzy subset 'J' is called a fuzzy subgroup (or fuzzy group) of G if, for all $x, y \in G$, it satisfies$(FG1): J(x, y) \geq min\{J(x), J(y)\}$ $(FG2): J(x^{-1}) = J(x)$.As a consequence, one shows that $J(e) = \max_{x \in G} J(x)$, so that the identity attains the maximum membership degree.

Example-2.11: Let $G = Z_4 = \{0, 1, 2, 3\}$ under addition modulo 4. Define a membership function $J: G \to [0,1]$ by

$$J(x) = \begin{cases} 1.0 & if\ x = 0 \\ 0.8 & if\ x = 2 \\ 0.6 & if\ x = 1,3 \end{cases}.$$

We can verify the two axioms of definition-2.10. Clearly, $(Z_4, .)$ forms a fuzzy subgroup of G.

3. M-fuzzy off group

In this section, we obtain the main results of this paper. A M-fuzzy off group extends of definition-2.10 by permitting membership values outside $[0,1]$, which preserving closure under operation and inversion invariance. As the main result of this article, we present the definition of M-fuzzy off group below.

Definition-3.1: Let $(G, ., e)$be a group and let $[\Delta, \delta]$ be an extended interval with $\Delta < 0 < 1 < \delta$.

A M-fuzzy off group of G is a mapping $J: M \times G \to [\Delta, \delta]$ which simultaneously,

(i) is a M-fuzzy off set in the definition of (M-fuzzy off set) (i.e.) there exists $x, y \in G$ with $J(mx) > 1$ and $J(my) < 0$.

(ii) Satisfies the usual M-fuzzy subgroup conditions;
$(FG1): J(mxy) \geq min\{J(x), J(y)\}$
$(FG2): J(mx^{-1}) = J(x)$.for all $x, y \in G$ and $m \in M$.

Example-3.2: Let $G = Z_4 = \{0, 1, 2, 3\}$ under addition modulo 4, and choose $[\Delta, \delta] = [-1.5, 1.6]$. Define, $J(mx) = \begin{cases} 1.6 & if\ x = 0 \\ 0.8 & if\ x = 1,3. \\ -1.5 & if\ x = 2 \end{cases}$

Then, if $m = 1, J(1.(1 + 3)) = J(0) = 1.6 \geq min\{J(1), J(3)\} \geq min\{0.8, 0.8\} \geq 0.8$.

$J(1.(-2)) = J(2) = -1.5$.

We have, $J(0) = \delta > 1$ and $J(2) = \Delta < 0$.

So, 'J' is a M-fuzzy off set and also satisfies $J(m(k + p)) \geq T\{J(k), J(p)\}$ and

$J(-k) = J(k)$. Hence J is a M-fuzzy off group of G.

Example-3.3: Let $G = Z_3 = \{0, 1, 2\}$ under addition modulo 3, and choose the extended

interval$[\Delta, \delta] = [-0.1, 1.5]$. Define, $J(mx) = \begin{cases} 1.5 & if\ x = 0 \\ -0.1 & if\ x = 1,2 \end{cases}$

Now we check the conditions of definition-3.1.

(i) Off set condition: $J(0) = 1.5 > 1$ and $J(1) = -0.1 < 0$, so J is a M-fuzzy off set.

(ii) For all $i, j \in G$

$$J\big(m(i+j)\big) = \begin{cases} J(0) = 1.5 & if\ i+j = 0 \\ J(1) = -0.1 & if\ i+j = 1 \\ J(2) = -0.1 & if\ i+j = 2 \end{cases}$$

Since $1.5 \geq -0.1$ and $-0.1 \geq -0.1$, we have $J\big(m(i+j)\big) \geq T\{J(i), J(j)\}$.

$J\big(-(mx)\big) = J(3-x) = J(x)$, for each $x \in \{1,2\}$ and $J(0) = J(0)$.

Thus 'J' defines a M-fuzzy off group on Z_3. Now we investigate the properties of the M-fuzzy off group. The following theorem holds;

Theorem-3.4: A mapping $J: M \times G \to [0,1]$ is a M-fuzzy subgroup of G if and only if it is a M-fuzzy off group whose image lies in $[0,1]$. In particular, every M-fuzzy subgroup is a special case of a M-fuzzy off group (with $\Delta = 0$ and $\delta = 1$).

Proof: If $J: M \times G \to [0,1]$ is a M-fuzzy subgroup of G, then trivially $J(G) \subseteq [0,1] = [\Delta, \delta]$ with $\Delta = 0$ and $\delta = 1$.

Since no element actually attains values outside the unit interval $[0,1]$, the 'off set' condition (existence of values >1 and <0) in vacuous when one sets $\Delta = 0$ and $\delta = 1$. Conditions on $J\big(m(xy)\big)$ and, $J(mx^{-1})$ coincide exactly with those of definition-3.1.

Conversely, if $J: M \times G \to [\Delta, \delta]$ is a M-fuzzy off group with $J(G) \subseteq [0,1]$, then the offset-specific requirements can be neglected, and the two equations

$J\big(m(x-y)\big) \geq T\{J(x), J(y)\}$ and $J(mx^{-1}) = J(x)$ are exactly the conditions of M-fuzzy subgroup. Hence, 'J' is M-fuzzy subgroup.

Theorem-3.5: Let $J: M \times G \to [\Delta, \delta]$ be a M-fuzzy off group. Then $J(me) = \max_{x \in G} J(mx)$.

In particular, $J(me) > 1$.

Proof: For any $x \in G$, since $x.x^{-1} = e$ and 'J' satisfies the M-fuzzy subgroup condition.

$$J(me) = J\big(m(x.x^{-1})\big) \geq T\{J(x), J(x^{-1})\}.$$

Hence, $J(me) \geq J(mx)$ for all x. So $J(me) = \max_{x \in G} J(mx)$. Because, '$J$' is M-fuzzy offset, there exists $x_0 \in G$ with $J(mx_0) > 1$, whence $J(me) > J(mx_0) > 1$.

Theorem-3.6: For any α with $\Delta < \alpha < J(me)$, define the α −cut $G_\alpha = \{x \in G / J(mx) \geq \alpha\}$.

Then, G_α is a (crisp) subgroup of G.

Proof: Since, $\alpha \leq J(me)$ and $J(me) \leq J(me)$, the identity element $e \in G_\alpha$. If $x, y \in G_\alpha$, then $J\big(m(x.y)\big) \geq T\{J(x), J(y)\} \geq T\{\alpha, \alpha\} \geq \alpha$ and $J(x^{-1}) \geq \alpha$.

Thus $x.y \in G_\alpha$ and $x^{-1} \in G_\alpha$. So G_α is closed under the group operation and inverses.

Theorem-3.7: Define the Cox of J by $Cox(J) = \{x \in G / J(mx) = J(me)\}$. Then $Cox(J)$ is a subgroup of G.

Proof: Because, $J(me) = \max_{x \in G} J(mx)$,

where $e \in Cox(J)$. If $x, y \in Cox(J)$, then

$$J\big(m(x.y)\big) \geq T\{J(x), J(y)\} = J(e),$$

and by maximality of $J(e)$ this forces $J\big(m(x.y)\big) = J(e)$.

Likewise , $J(mx^{-1}) = J(mx) = J(e)$.

Hence, $Cox(J)$ is closed under products and inverses.

Theorem-3.8: Let $J_1, J_2: M \times G \to [\Delta, \delta]$ be a M-fuzzy off groups such that there exists $a, b \in G$ with $J_1(ma).J_2(ma) > 1$ and $J_1(mb).J_2(mb) < 0$. Define $J(mx) = T\{J_1(x), J_2(x)\}, x \in G$. Then, '$J$' is also a M-fuzzy off group on G.

Proof: For all $x, y \in G$,

$$J\big(m(x.y)\big) = T\{J_1\big(m(x.y)\big), J_2\big(m(x.y)\big)\} \geq T\{T\{J_1(x), J_1(y)\}, T\{J_2(x), J_2(y)\}\} = T\{J(x), J(y)\}$$ and

$$J(mx^{-1}) = T\{J_1(mx^{-1}), J_2(mx^{-1})\} = T\{J_1(x), J_2(x)\}$$

Moreover, since $J_1(ma).J_2(ma) > 1$ and $J_1(mb).J_2(mb) < 0$, we have $J(ma) > 1$ and $J(mb) < 0$, so the off set conditions hold. Thus 'J' satisfies all the necessities of

definition-3.1.

4. M-fuzzy over group and M-fuzzy under group

In this section, the definition of the M-fuzzy over group and M-fuzzy under group are also presented below. These are subclasses of the M-fuzzy off group.

Definition-4.1: Let $(G, ., e)$ be a group and let $[0, \delta]$ be an extended interval with $0 < 1 < \delta$.

A M-fuzzy over group on G is a mapping $J: M \times G \to [0, \delta]$ which simultaneously,

(i) is an over M-fuzzy set, (i.e) there exists $x \in G$ with $J(mx) > 1$.

(ii) satisfies the M-fuzzy subgroup conditions;

1. $J\big(m(x.y)\big) \geq T\{J(x), J(y)\}$,
2. $J(mx^{-1}) = J(x), \forall x, y \in G$.

Thus a M-fuzzy over group is the extension of a M-fuzzy subgroup by allowing membership values above 1.

Example-4.2: Let $G = Z_3 = \{0, 1, 2\}$ under addition modulo 3, and choose the extended interval $[0, \delta] = [0, 1.6]$. Define, $J(mx) = \begin{cases} 1.6 & if\ x = 0 \\ 0.7 & if\ x = 1,2 \end{cases}$

We check the conditions of definition-4.1,

(i) over fuzzy condition: $J(0) = 1.6 > 1$, so, there exists an element with membership above 1.

(ii) $1 + 2 = 0; J(0) = 1.6 \geq T\{J(1), J(2)\} = T\{0.7, 0.7\} = 0.7$

$2 + 2 = 1; J(1) = 0.7 \geq T\{J(2), J(2)\} = T\{0.7, 0.7\} = 0.7$

$1 + 1 = 2; J(2) = 0.7 \geq T\{J(1), J(1)\} = T\{0.7, 0.7\} = 0.7$

For each $x \in \{1,2\}$, $-x = (3, -x)(mod\ 3)$, $J(-x) = J(3 - x) = 0.7 = J(x)$.

Hence. 'J' defines a M-fuzzy over group on G.

Theorem-4.3: Let $J: M \times G \to [0, \delta]$ be a M-fuzzy over group on G. Then

$J(me) = J(e) = \max_{x \in G} J(x)$ and in particular $J(e) > 1$.

Proof: For any $x \in G$, using $x.x^{-1} = e$ and the subgroup condition,

$$J(me) = J\big(m(x.x^{-1})\big) \geq T\{J(x), J(x^{-1})\} = J(x).$$

Hence, $(me) \geq J(mx)\ \forall x$, so equality holds as the supremum. Since 'J' is an over M-fuzzy set, $\exists\ x_0 \in G$ with $J(mx_0) > 1$, whence $J(me) > J(mx_0) > 1$.

Theorem-4.4: For any β with $1 < \beta \leq J(e)$, define the 'β'-cot $G_\beta = \{x \in G / J(mx) \geq \beta\}$. Then G_β is a (crisp) subgroup of G.

Proof: Since, $\beta < J(me)$, we have $e \in G_\beta$.

If $x, y \in G_\beta$, we have $J\big(m(x.y)\big) \geq T\{J(x), J(y)\} \geq T\{\beta, \beta\} = \beta$

Thus, $x.y \in G_\beta$ and $x^{-1} \in G_\beta$.

So, G_β is closed under the group operation and inverses.

Theorem-4.5: Let $J_1, J_2: M \times G \to [0, \delta]$ be a M-fuzzy over groups, each with the element greater than unity. Define $J(mx) = T\{J_1(x), J_2(x)\}, x \in G$. Then, '$J$' is also M-fuzzy over group of G.

Proof: Since each J_i has some r_i with $J_i(mr_i) > 1$, their minimum still satisfies $J(mr) > 1$ for 'r' chosen approximately.

Moreover, for all $x.y \in G$

$$J\big(m(x.y)\big) = T\{J_1\big(m(x.y)\big), J_2\big(m(x.y)\big)\} \geq T\{T\{J_1(x), J_1(y)\}, T\{J_2(x), J_2(y)\}\} = T\{J(x), J(y)\}$$ and

$$J(mx^{-1}) = T\{J_1(mx^{-1}), J_2(mx^{-1})\} = T\{J_1(x), J_2(x)\}$$

Thus, J satisfies all necessities of definition-4.1.

Theorem-4.6: Let $J: M \times G \to [\Delta,1]$ be a M-fuzzy under group on G, then $J(me) = \max_{x \in G} J(x)$.

Proof: Exactly as in theorem-4.3, using $x.x^{-1} = e$ and noting $J(me) \leq 1$ by the range.

Theorem-4.7: For any λ with $\Delta < \lambda < J(e)$, define $G_\lambda = \{x \in G / J(mx) \geq \lambda\}$, then G_λ is a (crisp) subgroup of G.

Proof: Similar to the proof of theorem-4.4.

Theorem-4.8: Let $J_1, J_2: M \times G \to [\Delta,1]$ be a M-fuzzy under groups, each with some element below 0. Define $J(mx) = T\{J_1(x), J_2(x)\}, \forall x \in G$, then J is also a M-fuzzy under group on G.

Proof: Since each J_i has some s_i with $J_i(ms_i) < 0$, their minimum still satisfies $J(ms) < 0$. The subgroup conditions follow just as in theorem-4.5.

Theorem-4.9: Let $(G, ., e)$ be a group and let $[\Delta,1]$ be an extended interval with $\Delta < 0 < 1$. A M-fuzzy under group on G is a mapping $J: M \times G \to [\Delta,1]$ which simultaneously,

(i) is an under M-fuzzy set, (i.e) there exists $y \in G$ with $J(my) < 0$.

(ii) satisfies the M-fuzzy subgroup conditions;

1. $J\big(m(x.y)\big) \geq T\{J(x), J(y)\}$,
2. $J(mx^{-1}) = J(x), \forall x, y \in G$.

Thus a M-fuzzy under group extends the notion of a M-fuzzy subgroup by allowing membership values below 0.

Example-4.10: Let $G = Z_3 = \{0, 1, 2\}$ under addition modulo 3, and choose the extended interval $[\Delta,1] = [-0.6, 1]$. Define, $J(mx) = \begin{cases} 1 & if\ x = 0 \\ -0.6 & if\ x = 1,2 \end{cases}$

Now we check the conditions of definition-4.1

(i) under M-fuzzy condition: $J(1) = J(2) = -0.6 < 0$, so, ∃ an element with membership below 0.

(ii) $1 + 2 = 0; J(0) = 1 \geq T\{J(1), J(2)\} \geq T\{-0.6, -0.6\} = -0.6$

$$2 + 2 = 1; J(1) = -0.6 \geq T\{J(2), J(2)\} \geq T\{-0.6, -0.6\} = -0.6$$

$$1 + 1 = 2; J(2) = -0.6 \geq T\{J(1), J(1)\} \geq T\{-0.6, -0.6\} = -0.6$$

5. Conclusion

In this paper, we studied the M-fuzzy off group, M-fuzzy over group, M-fuzzy under group and level cut and its characterizations.

6. Future work

Further research will develop more concrete mathematical structures like hyper groups and bipolar fuzzy subgroup.

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Takaaki Fujita1,, Arif Mehmood Khattak, Arkan A. Ghaib , “A Brief Study on Fuzzy Off-Group Theory”, Prospects for Applied Mathematics and data Analysis (PAMDA) Vol. 05, No. 01, PP. 1320-1326, 2025

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8. Author Declaration

* We confirm that the manuscript has been read and approved by all named authors and that there are no other persons who satisfied the criteria for authorship but are not listed. We further confirm that the order of authors listed in the manuscript has been approved by all of us.

***Conflict of Interest:** The authors declare no conflict of interest.