Non-Split Total Domination Number and Inverse Non-Split Total Domination Number of Some Special Graphs

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ABSTRACT:

A set $\mathbf{D}$ is said to be a dominating set if every vertex $\boldsymbol{v} \in \boldsymbol{V}-\boldsymbol{D}$ is adjacent to some vertex in $\mathbf{D}$. The minimum cardinality of the dominating set is the domination number denoted as $\gamma(G)$. A dominating set D is said to be a non-split dominating set if $\langle V-D\rangle$ is connected. The non-split domination number is denoted by $\gamma_{ns}(G)$.

A set $\boldsymbol{D_t}$ of vertices in a graph G is called the total dominating set if every vertex $v \in V$ is adjacent to an element of $\boldsymbol{D_t}$. The minimum cardinality of the total dominating set is the total domination number is denoted by $\gamma_t(G)$.

In this paper a new parameter, Non-split total dominating set, the non-split total domination number $\gamma_{nst}(G)$ and also the inverse non-split total domination number have been introduced. A total dominating set D_t is said to be a non-split total dominating set if $\langle V-D_t\rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Further a total dominating set D_t . If the complement set $V-D_t$ contains its own total D_t' and the induced subgraph $\langle V-D_t'\rangle$ is connected, then D_t' is an inverse non-split total dominating set. The minimum cardinality of such a set is the inverse non-split total domination number, denoted as $\gamma_{nst}'(G)$. In this paper we have obtained Non-split total domination number and an Inverse Non-split total domination number of some special graphs.

Keywords: Dominating set, Inverse dominating set, Total dominating set, Inverse total dominating set, Non-Split total dominating set, Inverse non-split total dominating set, Non-split total domination number and Inverse non-split total domination number

I. Introduction

In 1962, Ore used the name "Dominating set" and "domination number". In 1977, Cockayne and Hedetniemi made an interesting and extensive survey of the results known at that time about dominating sets in graphs. The survey paper of cockayne and Hedetniemi has generated a lot of interest in the study of domination in graphs. Domination has a wide range of applications in Radio station, modeling social networks, coding theory, nuclear power plants problems [21].

A non-empty subset D of vertices in a graph $G = (V, E)$ is a "Dominating set", if every vertex in $V - D$ is adjacent to some vertex in D. The domination number $\gamma(G)$ of G is the minimum cardinality of a minimal dominating set. A total dominating set D_t of G is a dominating set such that the induced subgraph $< D >$ has no isolated vertices. The total domination number $\gamma_t(G)$ of G is the minimum cardinality of a minimal total dominating set of G. Let D be the minimum dominating set of G. If $V - D$ contains a dominating set say D', then D' is called an inverse dominating set with respect to D. The inverse domination number $\gamma'(G)$ of G is the order of the smallest inverse dominating set of G. "Kulli and Sigarkandi" introduced the concept of inverse domination in Graphs. Let $G = (V, E)$ be a simple graph. Let D_t be the minimum total dominating set of G. If V- D_t contains a total dominating set D_t' of G, then D_t ' is called an inverse total dominating set with respect to D_t. The inverse total domination number $\gamma_t'(G)$ of G is the minimum number of vertices in an inverse total dominating set of G. The concept of total domination is first introduced by Cockayne, Dawes and Hedetniemi in 1980. V.R. Kulli and RadhaRajamani Iyer introduced the concept of inverse total domination in graphs [21].

A total dominating set D_t is said to be a split total dominating set if $\langle V - D_t \rangle$ is disconnected. The split total domination number is denoted by $\gamma_{st}(G)$. "T. Brindha and R. Shbiksha" introduced the concept of split total domination in graphs. In this paper, we found Non-split total domination number and Inverse Non-split total domination number of some special graphs.

II. Preliminaries

Definition 2.1

The **"Paley graph"** is a 6-regular graph on 13 vertices and 39 edges. It is also known as the Raymond Paley graph [21].

Definition 2.2

The **"Shrikhande graph"** is a named graph discovered by S.S.Shrikhande in 1959. It is a strongly regular graph with 16 vertices and 48 edges, with each vertex having degree 6 [21].

Definition 2.3

The **"Clebsch graph"** is a named graph discovered by Alfred Clebsch in 1868. It is a Strongly regular quintic graph on 16 vertices and 40 edges. The Clebsch graph is also known as Greenwood-Gleason graph. It is a 5-regular graph [21].

Definition 2.4

An **"Andrasfai graph"** is a triangle-free circulant graph named after Bela Andrasfai. It is also known as Wagner graph with each vertex having degree 6 [21].

Definition 2.5

The **"Chvatal graph"** is an undirected graph with 12 vertices and 24 edges, discovered by Vaclav Chvatal in 1970. It is a 4-regular graph [21].

Definition 2.6

The **"Soifer graph"** is a planar graph on 9 vertices and 20 edges [21].

Definition 2.7

The **"Moser graph"** (also called Mosers' Spindle graph) is an undirected graph with 7 vertices and 11 edges, named after the Mathematician Leo Moser and his brother William [21].

Definition 2.8

The "Franklin graph" is a 3-regular graph on 12 vertices and 18 edges. It is discovered by Philip Franklin [21].

Definition 2.9

The **"Golomb graph"** is a named graph discovered by Solomon w.Golomb . It is also known as polyhedral graph with 10 vertices and 18 edges [21].

Definition 2.10

The **"Herschel graph"** is a bipartite undirected graph with 11 vertices and 18 edges. It is discovered by British astronomer Alexander Stewart Herschel [21].

Definition 2.11

The **"Poussin graph"** is a planar graph on 15 vertices and 39 edges. It is discovered by Charles Jean de la ValleePoussin [21].

Definition 2.12

The **"Mobius-Kantor graph"** is a symmetric bipartite cubic graph on 16 vertices and 24 edges. It is discovered by August Ferdinand Mobius and Seligmann Kantor [21].

Definition 2.13

The **"Icosahedral graph"** is the Platonic graph which has 12 vertices and 30 edges. Since the Icosahedral graph is regular and Hamiltonian, it is also called Tayor graph [21].

Definition 2.14

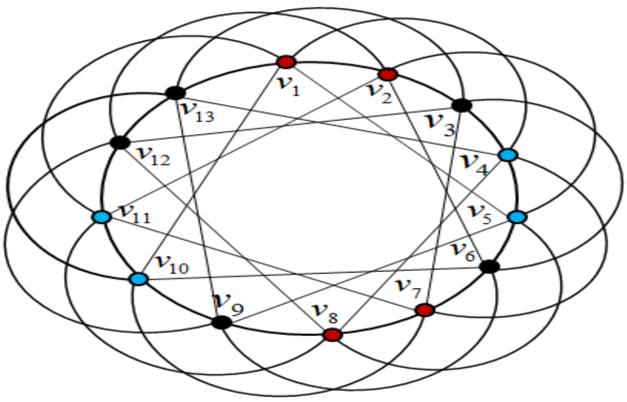
An **"Octahedron graph"** is a polyhedron graph with 8 faces,12 edges and 6 vertices. It is a 4-regular graph [21].

III. Results on Non-split total domination number and Inverse Non-split total domination number of some special graphs

Theorem 3.1

If G is a Paley graph, then $\gamma_{nst}(G) = \gamma'_{nst}(G) = 4$.

Proof:



Paley graph

From the figure, $V = \{v_1, v_2, \dots\dots, v_{12}, v_{13}\}$

(i) $D = \{v_1, v_3, v_8\} \implies \gamma(G) = 3$

(ii) $D_t = \{v_1, v_2, v_7, v_8\} \implies \gamma_{nst}(G) = 4$

(iii) $D_t{}' = \{v_4, v_5, v_{10}, v_{11}\} \implies \gamma'_{nst}(\mathrm{G}) = 4$

Step (i)

Let $G = (V, E)$ be a Paley graph with 13 vertices of degree 6. Let $V = \{v_1, v_2, \dots\dots, v_{12}, v_{13}\}$ be the vertex set of the graph G. To have a dominating set D, it is found that, the vertices $\{v_1, v_3, v_8\}$ forms a minimum dominating set. Thus, $|D| = 3$ .Hence $\gamma(G) = 3$.

Step (ii)

Now, the total dominating set $D_t = \{v_1, v_2, v_7, v_8\}$ is said to be a non-split total dominating set where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 4$. Hence $\gamma_{nst}(G) = 4$.

Step (iii)

Further a non-split total dominating set D_t, If the complement set $V - D_t$ contains its own total D_t' and the induced subgraph $\langle V - D_t' \rangle$ is connected, and then D_t' is an inverse non-split total dominating set $|D_t'| = |D_t|$. the minimum cardinality of such a set is the inverse non-split total domination number is denoted by $\gamma_{nst}'(G)$. Giving $|D_t'| = 4$. Hence $\gamma_{nst}'(G) = 4$.

Observation 3.1.1

In a Paley graph,

(i) $\gamma(G) < \gamma_{nst}(G)$ and (ii) $\gamma_{nst}(G) = \gamma_{nst}'(G)$

Theorem 3.2

If G is a Shrikhande graph with 16 vertices, then $\gamma(G) = 4 = \gamma_{nst}(G) = \gamma_{nst}'(G)$

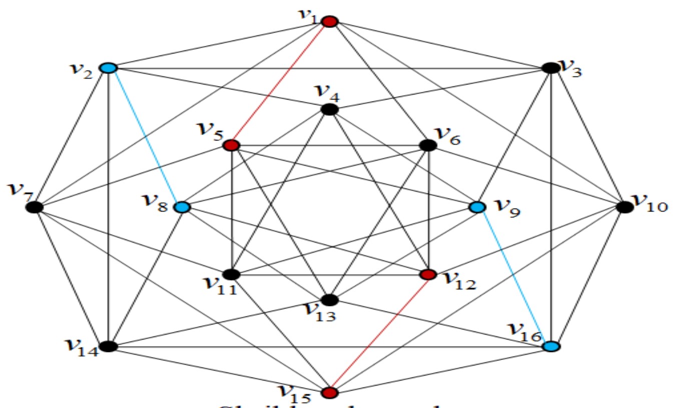
Proof:

From the figure, $V = \{v_1, v_2, \dots\dots, v_{15}, v_{16}\}$

(i) $D = \{v_1, v_5, v_{12}, v_{15}\} \Longrightarrow \gamma(G) = 4$

(ii) $D_t = \{v_1, v_5, v_{12}, v_{15}\} \Longrightarrow \gamma_{nst}(G) = 4$

(iii) $D_t' = \{v_2, v_8, v_9, v_{16}\} \Longrightarrow \gamma'_{nst}(G) = 4$



Shrikhande graph

(i) Consider the Shrikhande graph G with 16 vertices and G is a $6-$ regular graph. Let $\{v_1, v_2, v_3, v_7, v_{10}, v_{14}, v_{15}, v_{16}\}$ be the outer vertices and $\{v_4, v_5, v_6, v_8, v_9, v_{11}, v_{12}, v_{13}\}$ be the corresponding inner vertices. In the above figure, the vertices v_5 and v_1 dominate the vertices $\{v_{11}, v_{13}, v_6, v_9, v_7, v_{10}, v_2, v_3\}$ and similarly the vertices v_{12} and v_{15} dominate the vertices $\{v_4, v_8, v_{14}, v_{16}\}$ Therefore, the minimum dominating set is $\{v_1, v_5, v_{12}, v_{15}\}$. Hence $\gamma(G) = 4$.

(ii) Here, the total dominating set $\{v_1, v_5, v_{12}, v_{15}\}$ is said to be a non-split total dominating set, where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 4$. Hence $\gamma_{nst}(G) = 4$.

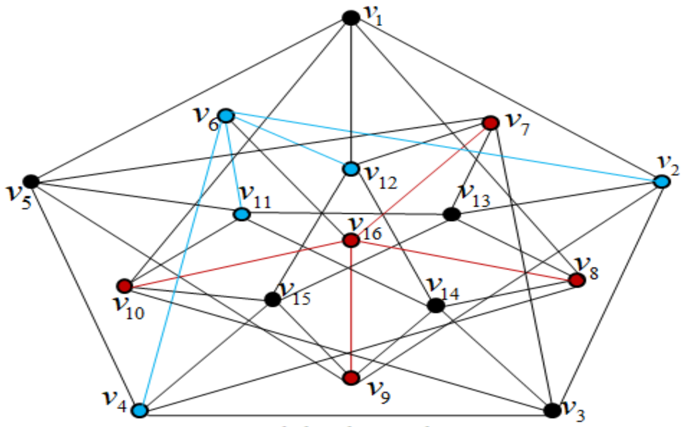
(iii) A set $D_t' \subseteq V - D_t$ is called an Inverse non-split total dominating set, If D_t' is itself a non-split total dominating set of G. The minimum cardinality of an inverse non-split total dominating set is called the inverse non-split total domination number is denoted by γ_{nst}'. Giving $|D_t'| = 4$. Hence $\gamma_{nst}'(G) = 4$.

Thus, In a Shrikhande graph, $(i)\ \gamma(G) = 4$ and $(ii)\ \gamma(G) = \gamma_{nst}(G) = \gamma_{nst}'(G)$

Theorem 3.3

If G is Clebsch graph, then $\gamma_{nst}(G) = 5 = \gamma_{nst}'(G)$

Proof:



Clebsch graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{15}, v_{16}\}$

(i) $D = \{v_7, v_8, v_9, v_{11}\} \Longrightarrow \gamma(G) = 4$

(ii) $D_t = \{v_7, v_8, v_9, v_{10}, v_{16}\} \Longrightarrow \gamma_{nst}(G) = 5$

(iii) $D_t' = \{v_6, v_4, v_2, v_{11}, v_{12}\} \Longrightarrow \gamma'_{nst}(G) = 5$

By the definition of Clebsch graph, the graph comprises of 16 vertices and G is a $5-$regular graph. Here, there are 4 vertices which dominate all the remaining vertices. Suppose $V = \{v_1, v_2, \dots \dots, v_{15}, v_{16}\}$ is the set of all vertices and $D = \{v_7, v_8, v_9, v_{11}\}$ is the minimum dominating set. Hence $\gamma(G) = 4$.

Consider the Clebsch graph with vertex set $V(G) = \{v_1, v_2, \dots \dots, v_{15}, v_{16}\}$. From the figure, the set $D_t = \{v_7, v_8, v_9, v_{10}, v_{16}\}$ is a total dominating set. Every vertex of G is adjacent to at least one vertex of D_t, and each vertex in D_t is adjacent to another vertex of D_t. Hence D_t Satisfies the conditions of a total dominating set.

To verify the non-split condition, consider the induced subgraph on $V - D_t = \{v_1, v_2, v_3, v_4, v_5, v_6, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}\}$. the vertices in $V - D_t$ remains connected. Since the subgraph induced by $V - D_t$ is connected, D_t is a non-split total dominating set. No non-split total dominating set with fewer than five vertices exists. Therefore, $\gamma_{nst}(G) = 5$.

Next, consider the set $D_t' = \{v_6, v_4, v_2, v_{11}, v_{12}\}$ which is contained in $V - D_t$. every vertex of the graph is adjacent to at least one vertex of D_t', and each vertex of D_t' has a neighbor within the set. Thus D_t' is a inverse non-split total dominating set of minimum cardinality 5. Therefore, $\gamma'_{nst}(G) = 5$.

NOTE:

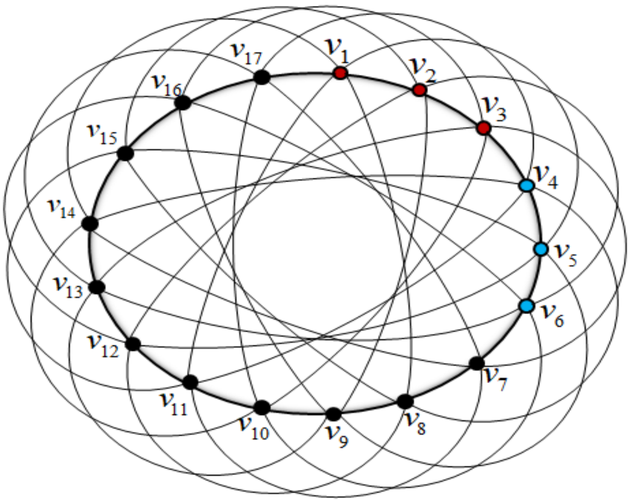
For the Clebsch graph,

$(i)\ \gamma(G) = 4$ and $(ii) \gamma(G) < \gamma_{nst}(G) = \gamma'_{nst}(G)$

Theorem 3.4

Let G be the Andrasfai graph with 17 vertices, then $\gamma(G) = \gamma_{nst}(G) = \gamma'_{nst}(G) = 3$

Proof:



Andrasfai graph

From the figure, $V = \{v_1, v_2, \ldots \ldots, v_{16}, v_{17}\}$

(i) $D = \{v_1, v_2, v_3\} \Rightarrow \gamma(G) = 3$

(ii) $D_t = \{v_1, v_2, v_3\} \Rightarrow \gamma_{nst}(G) = 3$

(iii) $D_t{}' = \{v_6, v_5, v_4\} \Rightarrow \gamma'_{nst}(G) = 3$

Let $G = (V, E)$ be a Andrasfai graph with 17 vertices of degree 6. Let $V = \{v_1, v_2, \ldots \ldots, v_{16}, v_{17}\}$ be the vertex set of the graph G. To have a dominating set D, it is found that, the vertices $\{v_1, v_2, v_3\}$ forms a minimum dominating set. Thus, $|D| = 3$ .Hence $\gamma(G) = 3$.

Now, the total dominating set $D_t = \{v_1, v_2, v_3\}$ is said to be a non-split total dominating set where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 3$. Hence $\gamma_{nst}(G) = 3$.

Further a non-split total dominating set D_t, If the complement set $V - D_t$ contains its own total D'_t and the induced subgraph $\langle V - D'_t \rangle$ is connected, and then D'_t is an inverse non-split total dominating set $|D'_t| = |D_t|$. the minimum cardinality of such a set is the inverse non-split total domination number is denoted by $\gamma'_{nst}(G)$. Giving $|D'_t| = 3$. Hence $\gamma'_{nst}(G) = 3$.

Observation 3.1.1

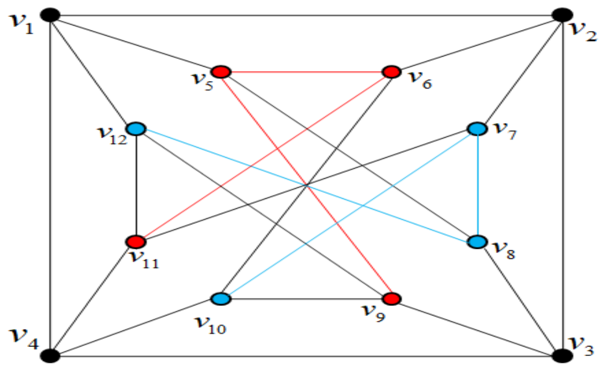
In a Andrasfai graph,

(i) $\gamma(G) = \gamma_{nst}(G) = 3$ and (ii) $\gamma_{nst}(G) = \gamma'_{nst}(G) = 3$

Theorem 3.5

If G is Chvatal graph, then $\gamma(G) = \gamma_{nst}(G) = \gamma'_{nst}(G) = 4$.

Proof:



Chvatal graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{11}, v_{12}\}$

(i) $D = \{v_1, v_3, v_5, v_7\} \Rightarrow \gamma(G) = 4$

(ii) $D_t = \{v_5, v_6, v_9, v_{11}\} \Rightarrow \gamma_{nst}(G) = 4$

(iii) $D_t{}' = \{v_7, v_8, v_{10}, v_{12}\} \Rightarrow \gamma'_{nst}(G) = 4$

(i) Consider the Chvatal graph G with 12 vertices and G is a $4-$ regular graph. Let $\{v_1, v_2, v_3, v_4\}$ be the outer vertices and $\{v_5, v_6, v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ be the corresponding inner vertices. In the above figure, To have a dominating set D, it is found that, the vertices $\{v_1, v_3, v_5, v_7\}$ forms a minimum dominating set. Thus, $|D| = 4$ .Hence $\gamma(G) = 4$.

(ii) Here, the total dominating set $D_t = \{v_5, v_6, v_9, v_{11}\}$ is said to be a non-split total dominating set, where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 4$. Hence $\gamma_{nst}(G) = 4$.

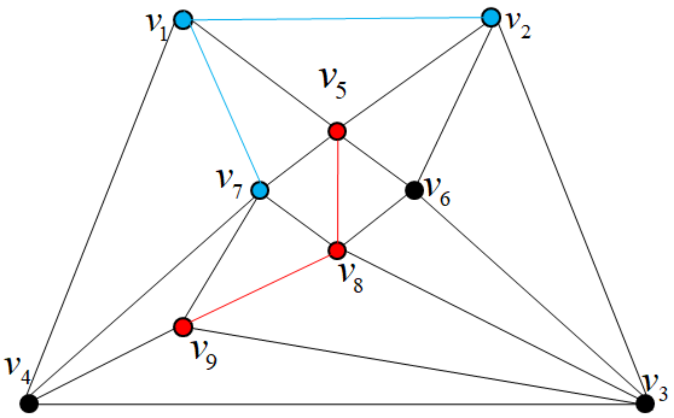
(iii) A set $D'_t \subseteq V - D_t$ is called an Inverse non-split total dominating set, If D'_t is itself a non-split total dominating set of G. The minimum cardinality of an inverse non-split total dominating set is called the inverse non-split total domination number is denoted by γ'_{nst}. Giving $|D'_t| = 4$. Hence $\gamma'_{nst}(G) = 4$.

Thus, In a Chvatal graph, $\gamma(G) = \gamma_{nst}(G) = \gamma'_{nst}(G) = 4$

Theorem 3.6

If G be the Soifer graph, then $\gamma_{nst}(G) = \gamma'_{nst}(G) = 3$

Proof:



Soifer graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_8, v_9\}$

(i) $D = \{v_1, v_8\}$ $\Longrightarrow \gamma(G) = 2$

(ii) $D_t = \{v_5, v_8, v_9\}$ $\Longrightarrow \gamma_{nst}(G) = 3$

(iii) $D_t{}' = \{v_1, v_2, v_7\}$ $\Longrightarrow \gamma'_{nst}(G) = 3$

By the definition of Soifer graph, the graph comprises of 9 vertices. Here, there are 2 vertices which dominate all the remaining vertices. Suppose $V = \{v_1, v_2, \dots \dots, v_8, v_9\}$ is the set of all vertices and $D = \{v_1, v_8\}$ is the minimum dominating set. Therefore $|D| = 2$. Hence $\gamma(G) = 2$.

Now, the set $D_t = \{v_5, v_8, v_9\}$ is a total dominating set. Every vertex of G is adjacent to at least one vertex of D_t, and each vertex in D_t is adjacent to another vertex of D_t. Hence D_t Satisfies the conditions of a total dominating set. Therefore $|D_t| = 3$.

To verify the non-split condition, consider the induced subgraph on $V - D_t = \{v_1, v_2, v_3, v_4, v_6, v_7\}$, the vertices in $V - D_t$ remains connected. Since the subgraph induced by $V - D_t$ is connected, D_t is a non-split total dominating set. No non-split total dominating set with fewer than three vertices exists. Hence $\gamma_{nst}(G) = 3$.

Next, consider the set $D_t{}' = \{v_1, v_2, v_7\}$ which is contained in $V - D_t$. every vertex of the graph is adjacent to at least one vertex of D_t' and each vertex of $D_t{}'$ has a neighbor within the set. Thus $D_t{}'$ is a inverse total dominating set. Now examine the induced subgraph on $V - D_t{}'$. This subgraph remains connected. Therefore, $D_t{}'$ satisfies the non-split condition. Hence $D_t{}'$ is an inverse non-split total dominating set with cardinality 3 .Therefore $|D_t'| = 3$. Hence $\gamma'_{nst}(G) = 3$

NOTE:

For the Soifer graph,

$(i)\ \gamma(G) = 2$ And $(ii) \gamma(G) < \gamma_{nst}(G) = \gamma'_{nst}(G) = 3$

Theorem 3.7

If G be the Moser graph with 7 vertices, then $\gamma'_{nst}(G) = 3$.

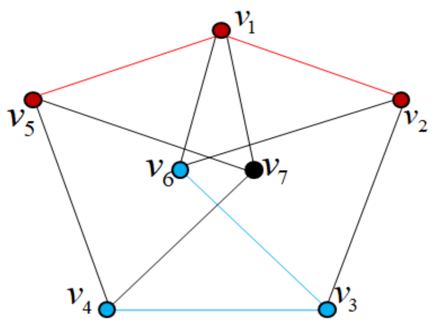
Proof:

From the figure, $V = \{v_1, v_2, \dots \dots, v_6, v_7\}$

(i) $D = \{v_2, v_5\}$ $\Longrightarrow \gamma(G) = 2$

(ii) $D_t = \{v_5, v_2, v_1\}$ $\Longrightarrow \gamma_{nst}(G) = 3$

(iii) $D_t{}' = \{v_3, v_4, v_6\}$ $\Longrightarrow \gamma'_{nst}(G) = 3$



Moser graph

Let G be the Moser graph with 7 vertices and 11 edges of degree 3. Let $V = \{v_1, v_2, \dots \dots, v_6, v_7\}$ be the set of all vertices in G. Now, the vertex v_2 is adjacent to the vertices $\{v_6, v_3, v_1\}$ and the vertex v_5 is adjacent to the vertices $\{v_4, v_7\}$ Therefore, the two vertices $\{v_2, v_5\}$ dominate the entire vertex set V. But it is not a total dominating set, because $\{v_2, v_5\}$are not adjacent in D. So, we include one more vertex v_1. So that $D_t = \{v_5, v_2, v_1\}$ is a total dominating set. Therefore, Every vertex in V is adjacent to atleast one vertex in D_t. Thus, the total dominating set $D_t = \{v_5, v_2, v_1\}$ is said to be a non-split total dominating set, where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 3$. Hence $\gamma_{nst}(G) = 3$. Also, there exists an inverse non-split total dominating set $D_t{}' = \{v_3, v_4, v_6\}$ which dominates the vertices of V with minimum cardinality 3. Thus $\gamma'_{nst}(G) = 3$

Note 3.7.1

In a Moser graph,

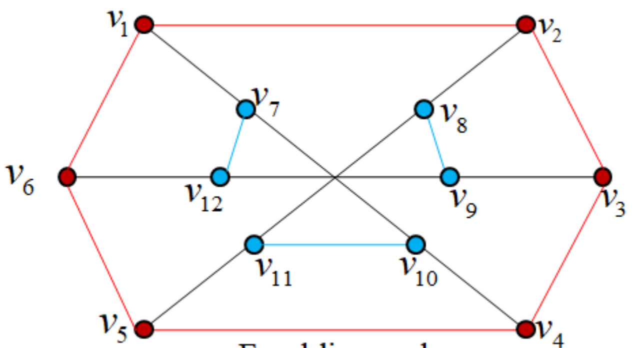
$(i)\ \gamma(G) = 2$ $\qquad$ $(ii)\ \gamma_{nst}(G) = 3$

$(iii) \gamma(G) < \gamma_{nst}(G) = \gamma'_{nst}(G)$

Theorem 3.8

If G be the Franklin graph, then $\gamma_{nst}(G) = 6 = \gamma'_{nst}(G)$

Proof:



Franklin graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{11}, v_{12}\}$

(i) $\quad D = \{v_1, v_4, v_7, v_8\} \quad \Longrightarrow \gamma(G) = 4$

(ii) $\quad D_t = \{v_1, v_2, v_3, v_4, v_5, v_6\} \quad \Longrightarrow \gamma_{nst}(G) = 6$

(iii) $\quad D_t{}' = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\} \quad \Longrightarrow \gamma'_{nst}(G) = 6$

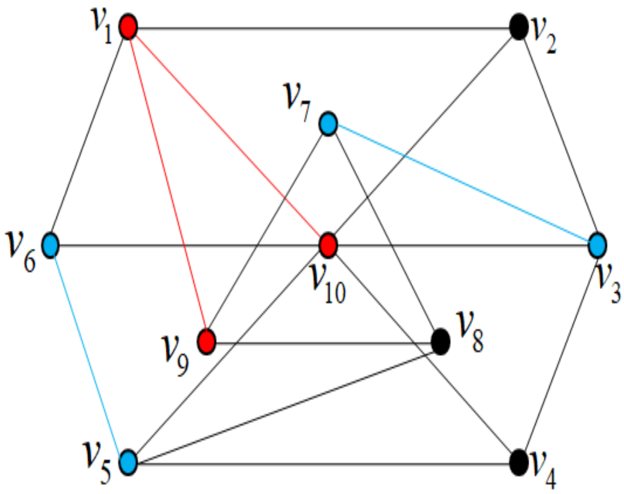
Let G be the Franklin graph with 12 vertices and G is a Cubic graph. Let $V = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ and $U = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ be the two subset of vertices of G. Here, the set of 4 vertices $\{v_1, v_4, v_7, v_8\}$ dominates the entire vertex set without isolated vertices.

Here, the set $D_t = \{v_1, v_2, v_3, v_4, v_5, v_6\}$ is the non-split total dominating set and $D_t{}' = \{v_7, v_8, v_9, v_{10}, v_{11}, v_{12}\}$ is the inverse non-split total dominating set. Hence $\gamma_{nst}(G) = 6 = \gamma'_{nst}(G)$.

Theorem 3.9

If G be the Golomb graph, then $\gamma_{nst}(G) = 3$ and $\gamma'_{nst}(G) = 4$.

Proof:



Golomb graph

From the figure, $V = \{v_1, v_2, \ldots \ldots, v_9, v_{10}\}$

(i) $D = \{v_7, v_{10}\}$ $\Longrightarrow \gamma(G) = 2$

(ii) $D_t = \{v_1, v_9, v_{10}\}$ $\Longrightarrow$ $\gamma_{nst}(G) = 3$

(iii) $D_t{}' = \{v_3, v_7, v_5, v_6\}$ $\Longrightarrow$ $\gamma'_{nst}(G) = 4$

By the definition of Golomb graph, the graph G comprises of 10 vertices. The set $D = \{v_7, v_{10}\}$ to dominate all the remaining vertices of V. suppose $V = \{v_1, v_2, \ldots \ldots, v_9, v_{10}\}$ is the set all vertices and $D = \{v_7, v_{10}\}$ is the minimum dominating set of G. Then, $\gamma(G) = 2$.

Here, the total dominating set $D_t = \{v_1, v_9, v_{10}\}$ is said to be a non-split total dominating set, where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 3$. Hence $\gamma_{nst}(G) = 3$.

Further, there exists an inverse non-split total dominating set $D_t{}' = \{v_3, v_7, v_5, v_6\}$ which dominates the vertices of V with minimum cardinality 4. Thus $\gamma'_{nst}(G) = 4$.

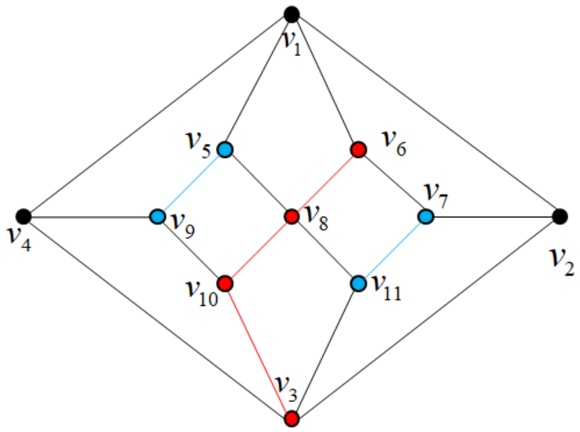
Note 3.9.1

In a Golomb graph , $(i)\gamma(G) < \gamma_{nst}(G)$ $(ii)\, \gamma_{nst}(G) < \gamma'_{nst}(G)$

Theorem 3.10

If G be the Herschel graph, then $\gamma_{nst}(G) = 4$ and $\gamma'_{nst}(G) = 4$

Proof:



Herschel graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{10}, v_{11}\}$

(i) $D = \{v_2, v_4, v_8\} \Rightarrow \gamma(G) = 3$

(ii) $D_t = \{v_3, v_6, v_8, v_{10}\} \Rightarrow \gamma_{nst}(G) = 4$

(iii) $D_t' = \{v_5, v_9, v_7, v_{11}\} \Rightarrow \gamma'_{nst}(G) = 4$

Let G be the Herschel graph of 11 vertices. Let $V = \{v_1, v_2, \dots \dots, v_{10}, v_{11}\}$ be the vertices of G. The subset $D = \{v_2, v_4, v_8\}$ dominates the entire vertex set V, and the set $\{v_2, v_4, v_8\}$ forms the minimum dominating set of G. Thus $\gamma(G) = 3$.In the above figure, the vertex subset $\{v_3, v_6, v_8, v_{10}\}$ forms the minimum non-split total dominating set D_t of G. Thus $\gamma_{nst}(G) = 4$. Further, there exists atleast one $D_t' = \{v_5, v_9, v_7, v_{11}\}$ such that $(V - D_t)$ contains an inverse non-split total dominating set. Hence $\gamma'_{nst}(G) = 4$.

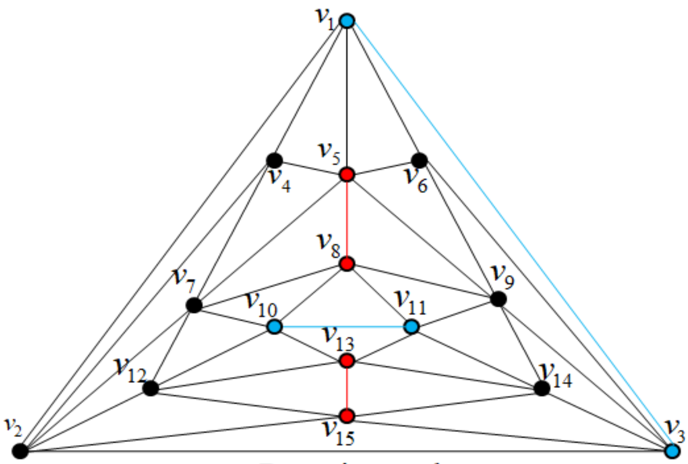
Note 3.10.1

In a Herschel graph , $(i)\,\gamma(G) < \gamma_{nst}(G)$ $(ii)\gamma_{nst}(G) = 4 = \gamma'_{nst}(G)$

Theorem 3.11

Let G be the Poussin graph, then $\gamma_{nst}(G) = \gamma'_{nst}(G) = 4$

Proof:



Poussin graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{14}, v_{15}\}$

(i) $D = \{v_1, v_8, v_{15}\} \Rightarrow \gamma(G) = 3$

(ii) $D_t = \{v_5, v_8, v_{13}, v_{15}\} \quad \Rightarrow \gamma_{nst}(G) = 4$

(iii) $D_t{}' = \{v_1, v_3, v_{10}, v_{11}\} \quad \Rightarrow \gamma'_{nst}(G) = 4$

Case (i): Let $G = (V, E)$ be the Poussin graph with 15 vertices. Let $V = \{v_1, v_2, \dots\dots, v_{14}, v_{15}\}$ be the vertex set of the graph G. The vertex subset D is a dominating set, since every vertex in $(V - D)$ is adjacent to some vertex in D. In the above figure, it is found that the set $D = \{v_1, v_8, v_{15}\}$ forms the minimum dominating set. Thus $|D| = 3$, Hence $\gamma(G) = 3$.

Case (ii): In Poussin graph, the set $D_t = \{v_5, v_8, v_{13}, v_{15}\}$ is a total dominating set. Every vertex of G is adjacent to at least one vertex of D_t, and each vertex in D_t is adjacent to another vertex of D_t. Hence D_t Satisfies the conditions of a total dominating set. Therefore $|D_t| = 4$.

To verify the non-split condition, consider the induced subgraph on $V - D_t = \{v_1, v_2, v_3, v_4, v_6, v_7, v_9, v_{10}, v_{11}, v_{12}, v_{14}\}$, the vertices in $V - D_t$ remains connected. Since the subgraph induced by $V - D_t$ is connected, D_t is a non-split total dominating set. No non-split total dominating set with fewer than four vertices exists. Therefore, $D_t = 4$. Hence $\gamma_{nst}(G) = 4$.

Case (iii): Next, consider the set $D_t{}' = \{v_1, v_3, v_{10}, v_{11}\}$ which is contained in $V - D_t$. every vertex of the graph is adjacent to at least one vertex of D'_t and each vertex of $D_t{}'$ has a neighbor within the set. Thus $D_t{}'$ is a inverse total dominating set. Now examine the induced subgraph on $V - D_t{}'$. this subgraph remains connected; therefore, $D_t{}'$ satisfies the non-split condition. Hence $D_t{}'$ is an inverse non-split total dominating set with cardinality 4. Therefore $|D'_t| = 4$. Hence $\gamma'_{nst}(G) = 4$.

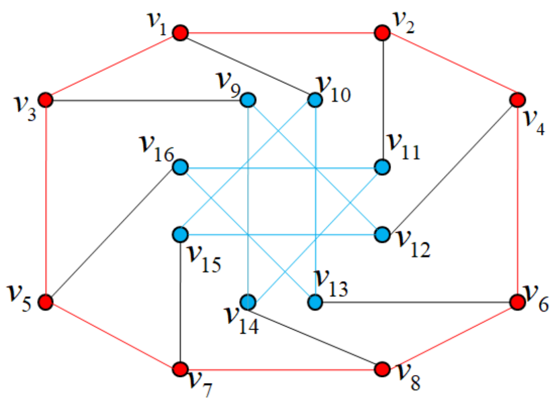
Note 3.11.1

In a Poussin graph, $\gamma_{nst}(G) = \gamma'_{nst}(G) = 4$

Theorem 3.12

Let G be the Mobius-Kantor graph with 16 vertices, then $\gamma_{nst}(G) = \gamma'_{nst}(G) = 8$.

Proof:



Mobius-Kantor graph

From the figure, $V = \{v_1, v_2, \dots\dots, v_{14}, v_{15}, v_{16}\}$

(i) $D = \{v_5, v_8, v_{10}, v_{11}, v_{12}\} \Rightarrow \gamma(G) = 5$

(ii) $D_t = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\} \Rightarrow \gamma_{nst}(G) = 8$

(iii) $D_t{}' = \{v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\} \Rightarrow \gamma'_{nst}(G) = 8$

Let G be a Mobius-Kantor graph with 16 vertices and G is a Cubic graph. Let $V = \{v_1, v_2, \dots \dots, v_{14}, v_{15}, v_{16}\}$ be the set of vertices of the graph. The vertex set V can be partitioned into two vertex subsets V_1 and V_2. $(i.e.) V_1 = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ and $V_2 = \{v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$ each one comprising of 8 vertices and the degree of each vertex being three. In the above figure, V_1 is taken as the minimum total dominating set $D_t = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ whose vertices dominates the remaining vertices, giving $\gamma_t(G) = 8$. Next, the total dominating set $D_t = \{v_1, v_2, v_3, v_4, v_5, v_6, v_7, v_8\}$ is said to be a non-split total dominating set, where $\langle V - D_t \rangle$ is connected and every vertex $v \in V$ is adjacent to an element of D_t. The minimum non-split total domination number is denoted by $\gamma_{nst}(G)$. Therefore $|D_t| = 8$. Hence $\gamma_{nst}(G) = 8$.

Further, there exists an inverse non-split total dominating set $D_t' = \{v_9, v_{10}, v_{11}, v_{12}, v_{13}, v_{14}, v_{15}, v_{16}\}$ which dominates the vertices of V with minimum cardinality 8. Thus $\gamma'_{nst}(G) = 8$.

Note 3.12.1

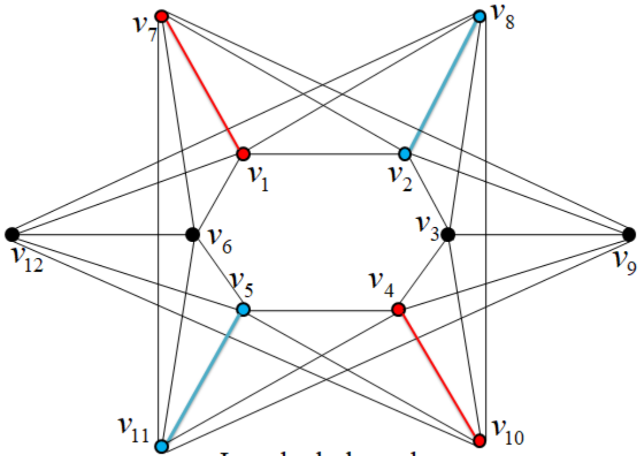
In a Mobius-Kantor graph, $(i)\ \gamma(G) = 5$ $(ii)\ \gamma_{nst}(G) = 8 = \gamma'_{nst}(G)$

$(iii)\ \gamma(G) < \gamma_{nst}(G) = \gamma'_{nst}(G)$

Theorem 3.13

Let G be the Icosahedral graph, then $\gamma_{nst}(G) = 4 = \gamma'_{nst}(G)$

Proof:



Icosahedral graph

From the figure, $V = \{v_1, v_2, \dots \dots, v_{12}\}$

(i) $D = \{v_7, v_{10}\} \Rightarrow \gamma(G) = 2$

(ii) $D_t = \{v_1, v_7, v_4, v_{10}\} \Rightarrow \gamma_{nst}(G) = 4$

(iii) $D_t' = \{v_2, v_8, v_{11}, v_5\} \Rightarrow \gamma'_{nst}(G) = 4$

Consider the Icosahedral graph G with 12 vertices of degree 5. Let $V = \{v_1, v_2, \dots \dots, v_{11}, v_{12}\}$ be the vertices of G. The subset $D = \{v_7, v_{10}\}$ dominates the entire vertex set V, and the set $\{v_7, v_{10}\}$ forms the minimum dominating set of G. Thus $\gamma(G) = 2$.

In the above figure, the vertex subset $\{v_1, v_7, v_4, v_{10}\}$ forms the minimum non-split total dominating set D_t of G. therefore $|D_t| = 4$. Thus $\gamma_{nst}(G) = 4$. Further, there exists atleast one $D_t' = \{v_2, v_8, v_{11}, v_5\}$ such that $(V - D_t)$ contains an inverse non-split total dominating set. Therefore $|D_t'| = 4$. Hence $\gamma'_{nst}(G) = 4$.

Note 3.13.1

In a Icosahedral graph, $(i)\ \gamma(G) = 2$

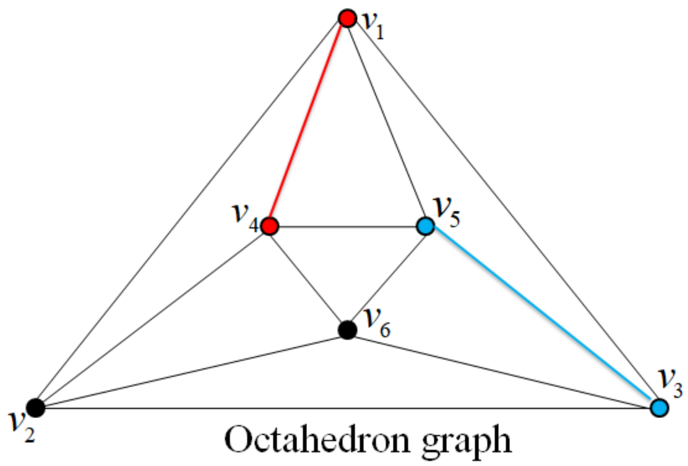
$(ii)\ \gamma_{nst}(G) = \gamma'_{nst}(G) = 4$

$(iii)\ \gamma(G) < \gamma_{nst}(G) = \gamma'_{nst}(G)$

Theorem 3.14

If G be the Octahedron graph, then $\gamma(G) = \gamma_{nst}(G) = \gamma'_{nst}(G) = 2$

Proof:



Octahedron graph

From the figure, $V = \{v_1, v_2, \dots\dots, v_6\}$

(i) $D = \{v_1, v_4\}$ $\Rightarrow \gamma(G) = 2$

(ii) $D_t = \{v_1, v_4\}$ $\Rightarrow \gamma_{nst}(G) = 2$

(iii) $D_t' = \{v_3, v_5\}$ $\Rightarrow \gamma'_{nst}(G) = 2$

Let G be the Platonic Solid Octahedron. Then G is a $4-$regular graph with 6 vertices and 12 edges. Let $D = \{v_1, v_4\}$ be the minimum dominating set and $D_t = \{v_1, v_4\}$ be the minimum non-split total dominating set and also $D_t' = \{v_3, v_5\}$ be the minimum inverse non-split total dominating set of G. In the above figure, the domination number as well as non-split total domination number and the inverse non-split total dominating number are the same. Hence $\gamma(G) = 2;\ \gamma_{nst}(G) = 2;\ \gamma'_{nst}(G) = 2.$

IV. Conclusion

In this paper, we have investigated the Non-split total domination number and the Inverse non-split total domination number of some special graphs. We further study on some more domination parameters of these special graphs.

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