

A Phase-Sensitive Ranking Framework for Complex Pythagorean Neutrosophic Fuzzy Sets with a Application to Renewable Energy Resource Selection

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Identifying renewable energy resources is a big challenge nowadays, since it is characterized by incomplete data, conflicting expert opinions, economic challenges, limitations and sometimes vary with time and phase. Classical fuzzy and intuitionistic fuzzy models include membership and non-membership values but fails to address phase valued details. This paper introduces a ranking framework that combines Complex Pythagorean Neutrosophic Fuzzy Numbers (CPNFNs) with the TOPSIS method. This method characterizes truth, indeterminacy, and falsity using phase and amplitude elements. This work establishes formal condition for boundedness, monotonicity and discrimination and illustrates, illustrates these with six-alternatives renewable energy selection problem. In addition, a comparative analysis is carried out to access the robustness and stability. This framework is designed for decision-making contexts where both uncertainty and periodic information play a significant role.

Keywords: Complex Pythagorean Neutrosophic Fuzzy Set; TOPSIS; MCDM ; renewable energy; phase-sensitive uncertainty.

Introduction

The transition toward renewable energy requires decisions that simultaneously consider technical performance, cost, environmental impact, social acceptance and reliability. Renewable-energy selection is therefore naturally formulated as a MCDM problem. Under realistic conditions, expert assessments are rarely completely precise: information may be incomplete, contradictory, hesitant or time-dependent.

Fuzzy set theory, introduced by Zadeh [1], provides a mathematical representation of graded membership. Atanassov's intuitionistic fuzzy sets added non-membership information [2]. Neutrosophic sets subsequently introduced

independent truth, indeterminacy and falsity components [3]. Pythagorean fuzzy sets broadened the admissible membership/non-membership domain through a quadratic constraint [4]. In parallel, complex fuzzy sets introduced complex-valued membership grades so that amplitude and phase can be represented simultaneously [6]. Yager and Abbasov demonstrated a connection between Pythagorean membership grades and complex numbers [5]. These advancements drive the current phase-sensitive extension. A significant methodological challenge is that merely combining existing uncertainty concepts does not inherently create novelty. To be considered publishable, a contribution must clearly demonstrate what is

innovative about the score function, why its axioms are beneficial, how it stands apart from existing score functions, and whether the resulting ranking is stable. This is especially crucial as recent studies have already explored intricate Pythagorean neutrosophic decision-making and weighted score functions [13,14].

Accordingly, the present study is positioned around four contributions: (i) a clearly specified CPNFN representation; (ii) a phase-sensitive additive ranking mechanism; (iii) integration with TOPSIS for renewable-energy selection; and (iv) explicit comparative and sensitivity validation.

Existing research directions provide important foundations for the development of the present study. Fuzzy sets [1] provide graded membership representation; however, they do not explicitly incorporate indeterminacy or phase information and therefore serve primarily as a foundational framework. Intuitionistic fuzzy sets [2] extend this representation by incorporating membership and non-membership information, but their ability to represent contradiction and phase information is limited, making them suitable as a foundation for the present study. Neutrosophic sets [3] introduce truth, indeterminacy, and falsity components, although their classical formulation is realvalued. Hence, they provide the neutrosophic foundation for the proposed framework. Pythagorean fuzzy sets [4] introduce a quadratic membership constraint, but they do not explicitly represent complex phase information and therefore contribute to the foundational development of the proposed approach. Complex fuzzy sets [6] incorporate both amplitude and phase information; however, they do not provide the neutrosophic truth, indeterminacy, and falsity structure required in the present study, and thus serve as the complex foundation.

Pythagorean neutrosophic sets [7] combine quadratic constraints with neutrosophic representation, but their classical formulation does not explicitly encode complex phase information. They therefore provide the neutrosophic foundation for extending the framework toward phase-sensitive

representations. More recent complex Pythagorean-neutrosophic decision models [13,14] integrate complex, Pythagorean, and neutrosophic information. Since existing studies already include weighted score and MCGDM formulations, a direct comparison with these approaches is required to establish the distinctiveness of the present framework. In this study, the proposed framework combines Complex Pythagorean Neutrosophic Fuzzy Sets (CPNFS) with phase-sensitive additive ranking and the TOPSIS method. The proposed contribution therefore needs to demonstrate distinct axioms and empirical stability in comparison with existing approaches.

Zadeh [1] introduced fuzzy sets to represent graded membership, describing membership in terms of a continuum of grades. Atanassov [2] extended the framework by assigning both membership and non-membership degrees. Smarandache [3] introduced neutrosophic sets with truth, indeterminacy and falsity components. Yager [4] proposed Pythagorean fuzzy subsets, replacing the intuitionistic linear constraint with a quadratic one. Yager and Abbasov [5] further connected Pythagorean membership grades with complex numbers. Ramot et al. [6] developed complex fuzzy sets using complex-valued membership functions.

Jansi et al. [7] studied Pythagorean neutrosophic sets and established a correlation measure for decision analysis. Zhang and Xu [8] extended TOPSIS to Pythagorean fuzzy MCDM. Hwang and Yoon [9] provide the classical TOPSIS foundation. Subsequent research has introduced a variety of Pythagorean, neutrosophic and complex extensions, including entropy- and score-function-based decision methods [10,11].

Palanikumar et al. [14] introduced a complex Pythagorean neutrosophic soft-set model and explicitly used weighted score functions with TOPSIS and VIKOR for multicriteria group decision-making. Rajalakshmi et al. [13] also applied complex neutrosophic sets to real-world decision-making. These studies indicate that the

present manuscript must distinguish its proposed ranking mechanism at the level of mathematical formulation and validation, rather than at the level of terminology alone [13,14].

2. Preliminaries and Definition

Definition 2.1: Fuzzy Set [16]

A fuzzy set A is defined by membership function μ_A that assigns each element $x \in X$ with a real number belongs to $[0,1]$

$$A = \{ \langle x, \mu_A(x) \rangle ; x \in X, 0 \leq \mu_A(x) \leq 1 \}$$

Definition 2.2: Intuitionistic Fuzzy Set (IFS) [2]

An IFS extends the fuzzy set in which the set is characterized by a membership μ_A and a non membership ν_A in $[0,1]$

$$A = \{ \langle x, \mu_A(x), \nu_A(x) \rangle ; x \in X, 0 \leq \mu_A(x) + \nu_A(x) \leq 1 \}$$

Definition 2.3: Neutrosophic Fuzzy set (NFS) [11]

A Neutrosophic fuzzy set is characterized by three degrees namely: Truth, Indeterminacy, Falsity (components are independent) defined by

$$A = \{ \langle x, \mu_A(x), \eta_A(x), \nu_A(x) \rangle ; x \in X, \mu_A(x) \in [0,1], \eta_A(x) \in [0,1], \nu_A(x) \in [0,1], 0 \leq \mu_A(x) + \eta_A(x) + \nu_A(x) \leq 3 \}$$

Definition 2.4 : Pythagorean Neutrosophic Fuzzy set (PNFS) [11]

A Pythagorean Neutrosophic Fuzzy set is an extension of NFS in which linearity is relaxed by introducing a square constraint and it is defined by

$$A = \{ \langle x, \mu_A(x), \eta_A(x), \nu_A(x) \rangle ; x \in X, 0 \leq \mu_A^2(x) + \eta_A^2(x) + \nu_A^2(x) \leq 2 \}$$

Definition 2.5 : Complex Pythagorean Neutrosophic Fuzzy set (CPNFS)

A Complex Pythagorean Neutrosophic Fuzzy set is represented by three complex valued membership functions namely: Truth, Indeterminacy, Falsity (components are independent) with the phase component. It is defined by

$$A = \{ \langle x, \mu_A(x), \eta_A(x), \nu_A(x) \rangle ; x \in X, 0 \leq \mu_A^2(x) + \eta_A^2(x) + \nu_A^2(x) \leq 2 \}$$

where $\mu_A(x) \in [0,1]$, $\eta_A(x) \in [0,1]$, $\nu_A(x) \in [0,1]$ are amplitude terms (real parts) represent the strength of truth, indeterminacy and falsity and $\phi_\mu(x), \phi_\eta(x), \phi_\nu(x) \in [0,2\pi]$ are phase terms (imaginary parts).

Definition 2.6: Score Function

For a CPNFS, a score function is defined as

$$S(A) = \frac{1}{2} [(T_A^2(x) + I_A^2(x) + F_A^2(x)) + \frac{1}{2\pi} (\phi_T - \phi_I - \phi_F)], S(A) \in [-1, 1]$$

Definition 2.7: Accuracy Function

For a CPNFS, an accuracy function is defined as

$$\alpha_A(x) = \frac{1}{2} [(\mu_A^2(x) + \eta_A^2(x) + \nu_A^2(x)) + (\phi_\mu - \phi_\eta - \phi_\nu)], \alpha_A(x) \in [0, 1]$$

Definition 2.8: Proposed Score Function

For a CPNFN, a proposed score function is defined as

$$\varnothing = \frac{1}{2} \left[\left(\frac{2T_A^2(x) - I_A^2(x) - F_A^2(x)}{2} \right) + \left(\frac{2\varphi_T^2 - \varphi_I^2 - \varphi_F^2}{8\pi^2} \right) \right], S(A) \in [-1, 1]$$

Definition 2.9: Proposed Accuracy Function

For a CPNFN, a proposed accuracy function is defined as

$$S(A) = \frac{1}{2} \left[\left(\frac{T_A^2(x) + I_A^2(x) + F_A^2(x)}{2} \right) + \left(\frac{\varphi_T^2 + \varphi_I^2 + \varphi_F^2}{12\pi^2} \right) \right], S(A) \in [0, 1]$$

Properties of the proposed score function Property 1: Boundary Condition :

A valid score function must typically bounded within the specific range [-1,1].

Let $\varnothing = \frac{1}{2} (M + P)$ where M and P are membership and phase component.

$$2^2 \varnothing - \varnothing -$$

$$\varnothing = \frac{\quad}{\quad}$$

$$2$$

$$- -$$

$$\varnothing = \frac{\quad}{\quad}$$

Membership Component

Under the PNFS constraint $T^2 + I^2 + F^2 \leq 2$, the maximum of $\varnothing$ is obtained at $T = 1, I = 0, F = 0$:

$$\varnothing = 2 \frac{\quad}{\quad} (1)^2 - 0 - = 1$$

The minimum is attained at $T = 0, I = 1, F = 1$:

$$2(0)^2 - 1 -$$

$$\varnothing = \frac{\quad}{\quad} = -1$$

Therefore, $-1 \leq \varnothing \leq 1$

Phase Component

The maximum is obtained at $\varnothing = , = , = :$

$$\varnothing - -$$

$$\varnothing = \frac{\quad}{\quad} = 1$$

The minimum is obtained at $\varnothing = , = , = :$

$$\varnothing - \varnothing - \varnothing$$

$$\varnothing = \frac{\quad}{\quad} = -1$$

Therefore, $-1 \leq \varnothing \leq 1$.

Overall Boundedness

Since $-1 \leq \varnothing \leq 1$, $-1 \leq \varnothing \leq 1$ their average also belongs to $[-1, 1]$.

$$-1 \leq \frac{S_M(A) + S_P(A)}{2} \leq 1$$

Hence the proposed score function is bounded in the interval $\phi \in [-1, 1]$.

Property 2 : Monotonicity (Directional Growth)

For the function to be logical, an increase in “Truth” must always lead to increase in the score. That is : as truth increases, the score must increase; as falsity increases, the score decreases.

$$\frac{\partial S}{\partial T} \geq 0$$

—

=

2

$$\frac{\partial S}{\partial F} \leq 0$$

Property 3: Indeterminacy Penalty

—

=

2

$$\frac{\partial S}{\partial I} \leq 0$$

Therefore, increasing the indeterminacy degree decreases the score. The proposed score incorporates an indeterminacy penalty.

Property 4: Phase Sensitivity The phase derivatives are:

$$\frac{\partial S}{\partial \theta_T} = -4 \frac{\partial S}{\partial \theta_F}$$

$$\frac{\partial S}{\partial \theta_T} = -8 \frac{\partial S}{\partial \theta_F}$$

$$\frac{\partial S}{\partial \theta_T} = -8 \frac{\partial S}{\partial \theta_F}$$

$$\frac{\partial S}{\partial \theta_T} = -8 \frac{\partial S}{\partial \theta_F}$$

$$\frac{\partial S}{\partial \theta_T} = -8 \frac{\partial S}{\partial \theta_F}$$

Thus, the score explicitly responds to phase information. Increasing truth phase increases the score, whereas increasing indeterminacy or falsity phase decreases it.

Property 5: Reduction Property

When all phase components vanish, $\theta_T = \theta_F = \theta_I = 0$ the phase term becomes zero.

$$\phi = 2 \frac{\partial S}{\partial T} - 4 \frac{\partial S}{\partial F}$$

Hence the complex phase-sensitive score reduces to its corresponding real-valued form.

Property 6: Boundedness of the Accuracy Function

For the membership component, the PNFS constraint gives:

$$2\phi + \phi + \phi$$

$$0 \leq \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \leq 1$$

2

For the phase component, because each phase lies in $[0, 2\pi]$

$$0 \leq \varphi_T + \varphi_I + \varphi_f \leq 2\pi$$

$$0 \leq \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \leq 1$$

The accuracy function is the average of these two normalized quantities. Therefore:

$$0 \leq (A) \leq 1 \quad \textbf{Property 7: Monotonicity - Accuracy Function}$$

$$= \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

2

1

$$= \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

2

$$= \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

2

$$= \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

12

$$\frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} = \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

12

$$= \frac{\varphi_T^2 + \varphi_I^2 + \varphi_f^2}{2} \geq 0$$

12

Thus, the accuracy measure increases with the magnitude of all membership and phase components. It should be interpreted as an overall information-magnitude measure, not as a falsity or indeterminacy penalty.

Property 8 : Comparison of two CPNFNs

Given two CPNFNs, μ_1 and μ_2 , the ranking order is as follows:

1. If $(\mu_1) > (\mu_2)$, then $\mu_1 > \mu_2$
2. If $(\mu_1) = (\mu_2)$, then calculate the accuracy function $(\mu_1) > (\mu_2)$, then $\mu_1 > \mu_2$

$(_1) < (_2)$, then $_1 < _2$

If both $(_1) = (_2)$ and $(_1) = (_2)$, then $_1 = _2$ ($_1$ and $_2$ are equivalent) **3. Methodology**

Selection and Ranking of Renewable Energy Resources Using TOPSIS with a Novel Score Function

The availability and efficient utilization of energy are essential for promoting economic development and achieving sustainability goals. Despite this, the extensive consumption of conventional fossil fuels has intensified environmental degradation by increasing greenhouse gas emissions and contributing to global warming and climate-related changes. In recent years, scientists and environmental researchers have increasingly emphasized the need to reduce the adverse impacts associated with conventional energy resources.

Developing countries, in particular, face significant challenges in meeting their growing energy demands while ensuring environmental sustainability. The adoption of renewable energy resources provides an effective approach to addressing these challenges. Renewable energy is essential for reducing energy crises, minimizing environmental degradation, and mitigating the impacts of climate change. Unlike fossil fuels, renewable energy resources are naturally replenished and are therefore considered sustainable and environmentally friendly.

Several renewable energy alternatives are available for meeting energy requirements, including

- **Wind energy** □ **Hydropower** □ **Solar energy** □ **Biomass energy**
- **Geothermal energy** □ **Tidal energy.**

However, selecting the most suitable energy resource among these alternatives is a complex problem, as it involves the simultaneous consideration of many criteria.

This research utilizes the TOPSIS method to prioritize and choose the most appropriate alternative energy sources. To enhance the

assessment and ranking of the renewable energy options, a newly suggested score function is integrated into the TOPSIS framework. This approach is designed to aid decision-makers in pinpointing the optimal renewable energy resource, thereby addressing energy shortages and fostering sustainable development alongside environmental conservation.

Let the alternatives be wind energy, hydropower, solar energy, biomass energy, geothermal energy, and tidal energy respectively as A_1, A_2, A_3, A_4, A_5 and A_6 ,

Step 1: Establish the Evaluation Criteria $\{C_1, C_2, C_3, C_4, C_5, C_6\}$

For a comprehensive evaluation of renewable energy resources, the following six criteria are considered:

- **C_1 (Energy Output):** The energy generation capacity and expected power output of the renewable energy resource (Benefit).
- **C_2 (Initial Cost):** The capital investment required for installation, infrastructure, equipment, and commissioning (Cost).
- **C_3 (Operating Cost):** The recurring expenditure associated with operation, maintenance, and management of the energy system (Cost).
- **C_4 (Environmental Sustainability):** The environmental performance of the energy resource, considering its sustainability and impact on the environment (Benefit).
- **C_5 (Reliability):** The ability of the renewable energy resource to provide a stable and dependable power supply over time (Benefit).
- **C_6 (Land/Space Requirement):** The amount of land or physical space required for establishing and operating the renewable energy system (Cost).

Criterion Classification

Criterion	Description	Weights
C ₁	Energy Output	0.20
C ₂	Initial Cost	0.15
C ₃	Operating Cost	0.15
C ₄	Environmental Sustainability	0.20
C ₅	Reliability	0.15
C ₆	Land/Space Requirement	0.15

These six criteria are used to evaluate the six renewable energy alternatives **A₁, A₂, A₃, A₄,**

A₅ and A₆, using the proposed Complex Pythagorean Neutrosophic Fuzzy (CPNF) TOPSIS approach. The truth, indeterminacy, falsity, and phase information associated with each alternative–criterion pair are represented using CPNFNs.

Step 2: Construct the CPNFN Decision Matrix

Experts assign a Complex Pythagorean Neutrosophic Fuzzy Number to each alternative A_i against each criterion C_j .

Step 3: Calculation of the Score Matrix (S_{ij})

Convert every CPNFN into a crisp score using your Proposed Additive Function:

$$() = 12 \frac{(\text{truth} + \text{indeterminacy} + \text{falsity} + \text{phase})}{4} = \frac{1}{4} [(2\text{truth} - 2\text{indeterminacy}) + (-\text{falsity} - \text{phase})]$$

Step 4: TOPSIS Ranking Procedure

1. Weighted Score matrix:

Since the proposed score function generates bounded scores in $[-1,1]$, no additional vector normalization is performed. The criterion weights are directly applied to obtain the weighted score matrix $\tilde{v}_i = \tilde{v}_i$.

2. Ideal Solutions:

- PIS (V^+): Maximize v_{ij} for benefit criteria and minimize for cost criteria.
- NIS (V^-): Minimize v_{ij} for benefit criteria and maximize for cost criteria.

2. Distance Calculation:

Use the Euclidean distance

$$+ = \sqrt{\sum (v_{ij} - v_j^+)^2} \quad \text{and} \quad - = \sqrt{\sum (v_{ij} - v_j^-)^2}$$

Step 5: Closeness Coefficient (CC_i)

Rank the energy sources by their closeness coefficient::

–

$$= \frac{d^-}{d^+ + d^-}$$

4. Numerical Example Construct the CPNFN Decision Matrix

C1 – Energy Output	CPNFN
A ₁	(0.90, 0.12, 0.12; 1.80, 0.40, 0.30)
A ₂	(0.86, 0.15, 0.16; 1.70, 0.45, 0.40)
A ₃	(0.92, 0.10, 0.10; 1.90, 0.30, 0.25)
A ₄	(0.74, 0.22, 0.20; 1.30, 0.70, 0.65)
A ₅	(0.82, 0.16, 0.15; 1.55, 0.45, 0.40)
A ₆	(0.84, 0.14, 0.13; 1.60, 0.35, 0.30)
C2 – Initial Cost	CPNFN
A ₁	(0.72, 0.20, 0.22; 1.30, 0.70, 0.80)
A ₂	(0.80, 0.15, 0.18; 1.10, 0.55, 0.70)
A ₃	(0.62, 0.24, 0.28; 1.00, 0.80, 0.90)
A ₄	(0.68, 0.22, 0.30; 1.15, 0.75, 0.95)
A ₅	(0.58, 0.28, 0.30; 0.90, 0.90, 1.00)
A ₆	(0.55, 0.30, 0.30; 0.85, 0.95, 1.00)
C3 – Operating Cost	CPNFN
A ₁	(0.78, 0.16, 0.18; 1.50, 0.50, 0.60)
A ₂	(0.84, 0.14, 0.16; 1.30, 0.45, 0.50)
A ₃	(0.66, 0.22, 0.25; 1.10, 0.70, 0.80)
A ₄	(0.72, 0.20, 0.25; 1.20, 0.65, 0.80)
A ₅	(0.62, 0.25, 0.28; 1.00, 0.80, 0.90)
A ₆	(0.52, 0.30, 0.34; 0.80, 0.95, 1.05)
C4 – Environmental Sustainability	CPNFN
A ₁	(0.88, 0.12, 0.10; 1.90, 0.30, 0.25)
A ₂	(0.82, 0.16, 0.15; 1.70, 0.40, 0.35)
A ₃	(0.90, 0.10, 0.09; 1.95, 0.25, 0.20)
A ₄	(0.70, 0.20, 0.22; 1.40, 0.55, 0.60)
A ₅	(0.78, 0.18, 0.18; 1.50, 0.45, 0.40)
A ₆	(0.76, 0.16, 0.17; 1.45, 0.40, 0.45)
C5 – Reliability	CPNFN
A ₁	(0.80, 0.15, 0.16; 1.60, 0.40, 0.40)
A ₂	(0.86, 0.12, 0.14; 1.80, 0.35, 0.30)

A₃	(0.90, 0.10, 0.10; 1.90, 0.25, 0.20)
A₄	(0.70, 0.20, 0.22; 1.30, 0.60, 0.65)
A₅	(0.94, 0.08, 0.08; 1.95, 0.20, 0.20)
A₆	(0.90, 0.10, 0.10; 1.85, 0.25, 0.25)
C₆ – Land/Space Requirement	CPNFN
A₁	(0.68, 0.20, 0.25; 1.20, 0.60, 0.80)
A₂	(0.75, 0.18, 0.20; 1.30, 0.50, 0.60)
A₃	(0.55, 0.28, 0.32; 0.90, 0.90, 1.00)
A₄	(0.78, 0.18, 0.18; 1.40, 0.50, 0.50)
A₅	(0.84, 0.14, 0.14; 1.60, 0.35, 0.30)
A₆	(0.72, 0.20, 0.22; 1.25, 0.60, 0.70)

Calculation of the Score Matrix (S_{ij})

Alternative	C₁	C₁	C₁	C₁	C₁	C₁
A₁	0.437252	0.251348	0.314334	0.425855	0.338371	0.217480
A₂	0.392082	0.316581	0.360039	0.358988	0.400989	0.280691
A₃	0.462955	0.161683	0.198244	0.447985	0.445072	0.104847
A₄	0.267326	0.204072	0.245084	0.243528	0.239349	0.309657
A₅	0.352307	0.124897	0.160458	0.314201	0.486253	0.374077
A₆	0.374752	0.103353	0.079209	0.299508	0.442555	0.251507

3. Weighted Normalized Matrix $v_{ij} = w_j S_{ij}$

Alternative	C₁	C₁	C₁	C₁	C₁	C₁
A₁	0.087450	0.037702	0.047150	0.085171	0.050756	0.032622
A₂	0.078416	0.047487	0.054006	0.071798	0.060148	0.042104
A₃	0.092591	0.024252	0.029737	0.089597	0.066761	0.015727
A₄	0.053465	0.030611	0.036763	0.048706	0.035902	0.046449
A₅	0.070461	0.018735	0.024069	0.062840	0.072938	0.056112
A₆	0.074950	0.015503	0.011881	0.059902	0.066383	0.037726

4. Positive and Negative Ideal Solutions

Benefit criteria: PIS uses the maximum value and NIS uses the minimum value. Cost criteria: PIS uses the minimum value and NIS uses the maximum value.

Positive Ideal Solution V^+ = (0.092591, 0.015503, 0.011881, 0.089597, 0.072938, 0.015727)

Negative Ideal Solution V^- = (0.053465, 0.047487, 0.054006, 0.048706, 0.035902, 0.056112)

5. Euclidean Separation Measures The separation measures are:

Alternative	d_i^+	d_i^-
A₁	0.050598	0.058308
A₂	0.064610	0.044044
A₃	0.020821	0.083156
A₄	0.079785	0.025991
A₅	0.054731	0.059860
A₆	0.041472	0.068202

6. Closeness Coefficient and Ranking

Alternative	d_i^+	d_i^-	CC_i	Rank
A₁	0.050598	0.058308	0.535396	3
A₂	0.064610	0.044044	0.405360	5
A₃	0.020821	0.083156	0.799753	1
A₄	0.079785	0.025991	0.245714	6
A₅	0.054731	0.059860	0.522378	4
A₆	0.041472	0.068202	0.621862	2

7. Final Ranking

A₃ is the best alternative because it has the highest closeness coefficient.

5. Comparative Analysis of Existing and Proposed CPNFN Ranking Methods

Comparison of Score Functions

Existing Score Function

For a CPNFN, a score function is defined as

$$(A) = \frac{1}{2} [(T_A^2(x) + I_A^2(x) + F_A^2) + \frac{1}{2\pi} (\varphi_T - \varphi_I - \varphi_F)], S(A) \in [-1, 1]$$

Proposed Score Function

For a CPNFN, a proposed score function is defined as

$$() = \frac{1}{2} \left[\frac{2T_A^2(x) - I_A^2(x) - F_A^2}{2} \right], (A) \in [-1, 1]$$

2. Comparative TOPSIS Results

Alternative	Existing	Rank	Proposed	Rank
A₁	0.557358	3	0.535397	3

A₂	0.453745	5	0.405360	5
A₃	0.824317	1	0.799753	1
A₄	0.213721	6	0.245714	6
A₅	0.505713	4	0.522378	4
A₆	0.616624	2	0.621862	2

Ranking obtained by both methods: $A_3 > A_6 > A_1 > A_5 > A_2 > A_4$.

3. Phase-Removal Comparison

Alternative	Full Proposed Method	Phase Removed
A₁	0.535397	0.534664
A₂	0.405360	0.391604
A₃	0.799753	0.796676
A₄	0.245714	0.260324
A₅	0.522378	0.532538
A₆	0.621862	0.633665

The phase-removal experiment shows that phase information changes the closeness coefficients, although the final ranking remains unchanged for the present dataset.

Discussion

The comparison demonstrates that the proposed score function is mathematically different from the existing CPNFN score while retaining the same overall ranking for the present renewable-energy dataset. The changes in closeness coefficients indicate that the proposed treatment of truth, indeterminacy, falsity, and phase information affects the discrimination among alternatives. In particular, A_3 remains the preferred alternative under both approaches. Since the ranking does not change in this dataset, the results should be interpreted as evidence of phase-sensitive numerical discrimination rather than proof of superior ranking performance. Further testing with different datasets and criterion-weight configurations would strengthen the validation.

The comparative study confirms that the proposed phase-sensitive score function produces different TOPSIS closeness coefficients from the existing CPNFN score while preserving the ranking $A_3 > A_6 > A_1 > A_5 > A_2 > A_4$ for the considered renewableenergy

problem. The phase-removal analysis further confirms that phase components influence the numerical evaluation of alternatives.

6. Conclusion

This study presents a phase-sensitive decision-making framework based on Complex Pythagorean Neutrosophic Fuzzy Numbers (CPNFNs) by integrating a proposed score function with the TOPSIS method. The score function incorporates both membership amplitudes and phase information, is bounded within $[-1,1]$, and satisfies the desired monotonicity properties. The proposed accuracy function, bounded within $[0,1]$ provides an additional measure of information magnitude.

The proposed framework was illustrated using a renewable energy resource selection problem that included six options and six criteria. The TOPSIS analysis resulted in a ranking where A_3 emerged as the top choice, having a closeness coefficient of 0.799753. These findings highlight the value of integrating phase information into the decisionmaking process. Future research could

explore larger datasets, incorporate multiple expert opinions, experiment with different weighting and aggregation methods, conduct

sensitivity analysis, and compare with other MCDM methods to further substantiate the framework.

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